ONLINE APPENDIX:
On the sources of information about latent variables in DSGE models

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A Evaluating the information and efficiency gain measures in linearized Gaussian models

Here I provide some details on how to evaluate the information and efficiency gains measures presented in Section 3 for the case of linearized Gaussian DSGE models. The reduced form solution of a typical linearized DSGE model can be represented as follows:

$$\begin{aligned} y_t &= s(\boldsymbol{\theta}) + C(\boldsymbol{\theta})x_t && \text{(A.1)} \\ x_t &= A(\boldsymbol{\theta})x_{t-1} + B(\boldsymbol{\theta})v_t, \quad x_0 = B_0(\boldsymbol{\theta})v_0 && \text{(A.2)} \\ v_t &\sim \mathcal{N}\left(0, \boldsymbol{I}_{n_v}\right), \quad w_t \sim \mathcal{N}\left(0, \boldsymbol{I}_{n_w}\right) && \text{(A.3)} \end{aligned}$$

where y_t is a n_y-dimensional vector of observed variables, w_t is n_w-dimensional vector of measurement errors, x_t is n_x-dimensional vector of state variables, v_t is n_v-dimensional vector of structural shocks, s is a n_y-dimensional vector, C is a $n_y \times n_x$ matrix, A is a $n_x \times n_x$ matrix, B and B_0 are $n_x \times n_v$ matrices.

A.1 Conditional distribution of z given y

We can write (A.1) and (A.2) in a stacked form as

$$\underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_T \end{pmatrix}}_{\boldsymbol{y}} = \underbrace{\begin{pmatrix} s \\ s \\ s \\ \vdots \\ s \end{pmatrix}}_{\boldsymbol{S}} + \underbrace{\begin{pmatrix} C & 0 & 0 & \dots & 0 \\ 0 & C & 0 & \dots & 0 \\ 0 & 0 & C & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & C \end{pmatrix}}_{\boldsymbol{C}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_T \end{pmatrix}}_{\boldsymbol{x}} \quad \text{(A.4)}$$

and

$$\underbrace{\begin{pmatrix} I & 0 & 0 & \dots & 0 \\ -A & I & 0 & \dots & 0 \\ 0 & -A & I & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & -A & I \end{pmatrix}}_{\boldsymbol{H}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_T \end{pmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{pmatrix} AB_0 & B & 0 & \dots & \dots \\ 0 & 0 & B & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & B \end{pmatrix}}_{\boldsymbol{L}} \underbrace{\begin{pmatrix} v_0 \\ v_1 \\ v_2 \\ \vdots \\ v_T \end{pmatrix}}_{\boldsymbol{v}} \quad \text{(A.5)}$$

Letting $\boldsymbol{M} = \boldsymbol{H}^{-1}\boldsymbol{L}$ (note that $\boldsymbol{H}$ is lower triangular matrix with ones on the main diagonal, thus invertible), we have

$$\begin{array}{rc} \boldsymbol{y} = & \boldsymbol{S}+\boldsymbol{C}\boldsymbol{x} \\ \boldsymbol{x} = & \boldsymbol{M}\boldsymbol{v} \\ \boldsymbol{v} = & \boldsymbol{v} \end{array} \Rightarrow \begin{pmatrix} \boldsymbol{y} \\ \boldsymbol{x} \\ \boldsymbol{v} \end{pmatrix} = \begin{pmatrix} \boldsymbol{S} \\ \boldsymbol{0}_{T\times n_x} \\ \boldsymbol{0}_{T\times n_v} \end{pmatrix} + \underbrace{\begin{pmatrix} \boldsymbol{C}\boldsymbol{M} \\ \boldsymbol{M} \\ \boldsymbol{I}_{T\times n_v} \end{pmatrix}}_{\boldsymbol{\Lambda}} \boldsymbol{v} \tag{A.6}$$

Note that $\boldsymbol{v} \sim \mathcal{N}(0, \boldsymbol{I}_{T\times n_w})$ and $\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Lambda}'$ is a singular matrix. Using the definition in Rao (2001, Chapter 8), $(\boldsymbol{y}', \boldsymbol{x}', \boldsymbol{v}')'$ has a singular normal distribution with mean $\left(\boldsymbol{S}', \boldsymbol{0}'_{T\times n_x}, \boldsymbol{0}'_{T\times n_v}\right)'$, and covariance matrix $\boldsymbol{\Sigma}$. Therefore, the marginal distribution of every subset of elements of $(\boldsymbol{y}', \boldsymbol{x}', \boldsymbol{v}')'$ is also singular normal distribution. In particular, collecting the unique elements of $(\boldsymbol{x}', \boldsymbol{v}')'$ into the vector $\boldsymbol{z}$, it follows that $(\boldsymbol{y}', \boldsymbol{z}')'$ is a vector with singular normal distribution. Moreover, the conditional distribution of $\boldsymbol{z}$ given $\boldsymbol{y}$ is singular normal, with mean $\mathrm{E}(\boldsymbol{z}|\boldsymbol{y}) = \boldsymbol{\Sigma}_{\boldsymbol{z}\boldsymbol{y}}\boldsymbol{\Sigma}^{\dagger}_{\boldsymbol{y}\boldsymbol{y}}(\boldsymbol{y}-\boldsymbol{S})$ and covariance matrix $\mathrm{V}(\boldsymbol{z}|\boldsymbol{y}) = \boldsymbol{\Sigma}_{\boldsymbol{z}^j|\boldsymbol{y}} = \boldsymbol{\Sigma}_{\boldsymbol{z}\boldsymbol{z}} - \boldsymbol{\Sigma}_{\boldsymbol{z}\boldsymbol{y}}\boldsymbol{\Sigma}^{\dagger}_{\boldsymbol{y}\boldsymbol{y}}\boldsymbol{\Sigma}_{\boldsymbol{y}\boldsymbol{z}}$, where $\boldsymbol{\Sigma}^{\dagger}_{\boldsymbol{y}\boldsymbol{y}}$ is the generalized inverse of $\boldsymbol{\Sigma}_{\boldsymbol{y}\boldsymbol{y}}$, and $\boldsymbol{\Sigma}_{\boldsymbol{r}\boldsymbol{c}}$ is the submatrix of $\boldsymbol{\Sigma}$ obtained by removing all row indices other than those corresponding to the elements of $\boldsymbol{r}$ and all column indices other than those corresponding to the elements of $\boldsymbol{c}$. Note that $\boldsymbol{z}$ contains the realizations of all – endogenous and exogenous – latent variables in the model. The conditional distribution of the realizations of a single latent variable $\boldsymbol{z}^j$ given $\boldsymbol{y}$ is derived analogously, by selecting the row and column indices of $\boldsymbol{\Sigma}$ that correspond to the indices of the elements of $\boldsymbol{z}^j$ in the vector $(\boldsymbol{y}', \boldsymbol{x}', \boldsymbol{v}')'$.

A.2 Fisher information matrix

From (A.6) we have that the marginal distribution of $\boldsymbol{y}$ is Gaussian with mean $\mathrm{E}\,\boldsymbol{y} = \boldsymbol{S}$ and covariance matrix $\boldsymbol{V}(\boldsymbol{y}) = \boldsymbol{\Sigma}_{\boldsymbol{y}} = \boldsymbol{C}\boldsymbol{M}\boldsymbol{M}'\boldsymbol{C}'$. The (k,l)-th element of Fisher information matrix is given by (see Kay (1993, Chapter 3.9) for proof)

$$\{\mathcal{I}\}_{k,l} = \frac{1}{2}\operatorname{tr}\left(\boldsymbol{\Sigma}_{\boldsymbol{y}}^{-1}\partial_k\boldsymbol{\Sigma}_{\boldsymbol{y}}\boldsymbol{\Sigma}_{\boldsymbol{y}}^{-1}\partial_l\boldsymbol{\Sigma}_{\boldsymbol{y}}\right) + \partial_k\boldsymbol{S}'\boldsymbol{\Sigma}_{\boldsymbol{y}}^{-1}\partial_l\boldsymbol{S} \tag{A.7}$$

where I have used $\partial_i X$ to denote the derivative of a matrix X w.r.t. θ_i. The required derivatives of $\boldsymbol{S}$ and $\boldsymbol{\Sigma}_{\boldsymbol{y}}$ with respect to the elements of $\boldsymbol{\theta}$ are straightforward to obtain once we have the derivatives of the matrices A, B, C and s from (A.1) and (A.2), which can be computed as shown in Iskrev (2008). For instance, the derivative of $\boldsymbol{\Sigma}_{\boldsymbol{y}}$ with respect to θ_i is

$$\partial_i\boldsymbol{\Sigma}_{\boldsymbol{y}} = \partial_i\boldsymbol{C}\boldsymbol{M}\boldsymbol{M}'\boldsymbol{C}' + \boldsymbol{C}\partial_i\boldsymbol{M}\boldsymbol{M}'\boldsymbol{C}' + \boldsymbol{C}\boldsymbol{M}\partial_i\boldsymbol{M}'\boldsymbol{C}' + \boldsymbol{C}\boldsymbol{M}\boldsymbol{M}'\partial_i\boldsymbol{C}' \tag{A.8}$$

The definition of $\boldsymbol{C}$ (see (A.4)) show how to obtain $\partial_i\boldsymbol{C}$ from $\partial_i C$. Furthermore, using that $\partial_i X^{-1} = -X^{-1}\partial_i X X^{-1}$, and $\boldsymbol{M} = \boldsymbol{H}^{-1}\boldsymbol{L}$ we have

$$\partial_i\boldsymbol{M} = -\boldsymbol{H}^{-1}\partial_i\boldsymbol{H}\boldsymbol{H}^{-1}\boldsymbol{L} + \boldsymbol{H}^{-1}\partial_i\boldsymbol{L} \tag{A.9}$$

which shows how to construct $\partial_i \boldsymbol{M}$ from the derivative of $\boldsymbol{H}$, which is a simple function of $A(\boldsymbol{\theta})$, on one hand, and that of $\boldsymbol{L}$, which is a simple function of $A(\boldsymbol{\theta}), B(\boldsymbol{\theta})$ and $B_0(\boldsymbol{\theta})$ (see (A.5)). Note that in (A.2) I have left $B_0(\boldsymbol{\theta})$, which determines the variance of the initial state x_0, unspecified. Typically it is assumed that the distribution of the initial state is the same as the unconditional distribution of x, in which case B_0 will be a simple function of A and B.

The asymptotic information matrix can be computed as in Whittle (1953) by using a frequency domain approximation of the likelihood function (see also Davies (1983)). The approximation involves replacing the covariance matrix of the joint distribution $\boldsymbol{\Sigma_y}$ with a circulant matrix, which can be diagonalized and is thus much cheaper to invert. As Whittle (1953) showed, the (k,l)-th element of the asymptotic information matrix for a multivariate Gaussian process with zero mean is:

$$\{\mathcal{I}_1\}_{k,l} = \frac{1}{4\pi}\int_{-\pi}^{\pi} \operatorname{tr}\left(\frac{\partial F(\omega)}{\partial\theta_k}F^{-1}(\omega)\frac{\partial F(\omega)}{\partial\theta_l}F^{-1}(\omega)\right), \tag{A.10}$$

where $F(\omega)$ is the spectral density matrix of the zero-mean process $y_t - s(\boldsymbol{\theta})$. $\{\mathcal{I}_1\}_{k,l}$ reflects information about $\boldsymbol{\theta}$ contained only in the second order moments of data. The information about $\boldsymbol{\theta}$ in the mean of y_t is given by (see Qu and Tkachenko (2012))

$$\{\mathcal{I}_2\}_{k,l} = \frac{1}{2\pi}\left(\frac{\partial s}{\partial\theta_k}\right)' F^{-1}(0)\left(\frac{\partial s}{\partial\theta_l}\right) \tag{A.11}$$

The full information matrix is $\mathcal{I} = \mathcal{I}_1 + \mathcal{I}_2$. To compute it we need the derivatives of $s = s(\boldsymbol{\theta})$ and $F(\omega) = F(\omega, \boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$. The derivative $\partial F(\omega)/\partial\theta_k$, can be obtained using that the spectral density matrix for the model in (A.1)-(A.2) is (see for instance Hansen and Sargent (2013))

$$F(\omega) = \frac{1}{2\pi}C\Psi^- BB'\Psi^+ C' \tag{A.12}$$

where $\Psi^- = (\boldsymbol{I}_{n_x} - A\exp(-i\omega))^{-1}$ and $\Psi^+ = (\boldsymbol{I}_{n_x} - A'\exp(i\omega))^{-1}$. Therefore, we have

$$\begin{aligned}\partial_i F(\omega) = \partial_i C\Psi^- BB'\Psi^+ C' + C\partial_i\Psi^- BB'\Psi^+ C' + C\Psi^-\partial_i BB'\Psi^+ C' +\\ C\Psi^- B\partial_i B'\Psi^+ C' + C\Psi^- BB'\partial_i\Psi^+ C' + C\Psi^- BB'\Psi^+\partial_i C'\end{aligned}$$

where

$$\partial_i\Psi^- = -\Psi^-\left(\boldsymbol{I}_m - \partial_i A\exp(-i\omega)\right)\Psi^- \tag{A.13}$$

and similarly for Ψ^+.

B Schmitt-Grohé and Uribe (2012) model

The model economy is populated by a continuum of identical agents each maximizing the following lifetime utility function

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \frac{\left[C_t - bC_{t-1} - \psi H_t^{\theta} S_t\right]^{1-\sigma} - 1}{1-\sigma}, \tag{B.1}$$

where ζ_t is a preference shock, C_t is consumption, H_t is hours worked, and S_t is a geometric average of past habit-adjusted consumption: $S_t = (C_t - bC_{t-1})^{\gamma} S_{t-1}^{1-\gamma}$. The household budget constraint is given by

$$C_t + A_t I_t + T_t = W_t H_t + r_t u_t K_t + P_t, \tag{B.2}$$

where A_t is a non-stationary investment specific productivity growing at rate μ_t^a. The variable T_t denotes lump-sum taxes, W_t is the wage rate, r_t is rental rate of capital, u_t is capacity utilization, K_t is capital stock, and P_t denotes profit. The law of motion for capital stock is

$$K_{t+1} = (1 - \delta(u_t))K_t + z_t^I I_t \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \mu^I\right)\right], \tag{B.3}$$

where I_t is investment, δ is the rate of depreciation – an increasing function of the rate of capacity utilization u_t, κ is a parameter that determines the convexity of the investment adjustment cost function, μ^I is is the steady state growth rate of investment, and z_t^I is a stationary investment specific productivity shock.

Final good Y_t is produced with the following production function:

$$Y_t = z_t(u_t K_t)^{\alpha_k}(X_t H_t)^{\alpha_h}(X_t L)^{1-\alpha_k-\alpha_h}, \tag{B.4}$$

where z_t is a stationary neutral productivity shock, L is a fixed factor of production,[1] and X_t is a non-stationary neutral productivity growing at rate μ_t^x.

The labor input H_t, which is used by final-good-producing firms, is obtained by combining differentiated labor services H_{jt} supplied by monopolistically competitive labor unions,

$$H_t = \left[\int_0^1 H_{jt}^{\frac{1}{1+\mu_t}} dj\right]^{1+\mu_t}, \tag{B.5}$$

where μ_t is a wage markup shock with steady state value $\mu > 1$.

Each period the government spends an amount G_t, financed with lump-sum taxes. G_t is determined exogenously and is assumed to grow at rate X_t^G, defined as a smoothed

[1] The fixed factor of production generates decreasing returns to scale in the two variable factors of production K_t and H_t. As shown by Jaimovich and Rebelo (2009) this allows for a positive response of the value of the firm to future expected increases in productivity.

version of the trend in Y_t, given by $X_t^Y = X_t A_t^{\alpha_k/(\alpha_k-1)}$.

Each of the seven shocks is driven by three independent innovations, two anticipated and one unanticipated. More precisely, the process governing shock x_t for $x = \mu^a, \mu^x, z^I, z, \mu, g, \zeta$ is given by

$$\ln(x_t/x) = \rho_x \ln(x_{t-1}/x) + \sigma_x^0 \varepsilon_{x,t}^0 + \sigma_x^4 \varepsilon_{x,t-4}^4 + \sigma_x^8 \varepsilon_{x,t-8}^8, \tag{B.6}$$

where $\varepsilon_{x,t}^j$ for $j = 0, 4, 8$ are independent standard normal random variables.

SGU report results based on estimation of the model using quarterly data on seven macroeconomic series: the growth rate of per capita real GDP ($y_t = \triangle \ln Y_t$) contaminated with a measurement error, the growth rates of real consumption ($c_t = 100\triangle \ln C_t$), real investment ($i_t = 100\triangle \ln A_t I_t$), real government expenditure ($g_t = 100\triangle \ln G_t$), and hours ($h_t = 100\triangle \ln H_t$), and the growth rates of the relative price of investment ($a_t = 100\triangle \ln A_t$) and of total factor productivity ($tfp_t = 100\triangle \ln TFP_t$).[2]

In addition to these variables, the model makes predictions about the behavior of two asset price variables: the value of the firm and the risk-free real interest rate. The value of the firm V^F can be computed as

$$V_t^F = Y_t - W_t H_t - A_t I_t + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^F, \tag{B.7}$$

where Λ_t is the Lagrange multiplier associated with the household's budget constraint. The risk-free real interest rate is given by

$$R_t = \frac{1}{\beta} \frac{\Lambda_t}{E_t \Lambda_{t+1}}. \tag{B.8}$$

In estimation, the value of the firm can be matched to stock price data. In particular, $v_t^f = \triangle \ln V_t^F$ can be represented with the growth rate of the real per capita value of the stock market. Similarly, data on $r_t = \log R_t$ can be obtained by deflating the nominal rate on the three-month Treasury bill by the inflation rate implied by the GDP deflator.

[2]The growth rate of total factor productivity in the model is given by $tfp_t = 100\left(\triangle \ln z_t + (1-\alpha_k)\ln \mu_t^x\right)$.

Table B1: Parameter values, SGU (2012) model

	parameter	MLE	posterior median
θ	Frisch elasticity of labor supply	5.39	4.74
γ	wealth elasticity of labor supply	0.00	0.00
κ	investment adjustment cost	25.07	9.11
δ_2/δ_1	capacity utilization cost	0.44	0.34
b	habit in consumption	0.94	0.91
ρ_{xg}	government spending	0.74	0.72
ρ_z	AR stationary neutral productivity	0.96	0.92
ρ_{μ^a}	AR non-stationary investment-specific productivity	0.48	0.48
ρ_g	AR government spending	0.96	0.96
ρ_{μ^x}	AR non-stationary neutral productivity	0.27	0.38
ρ_μ	AR wage markup	0.98	0.98
ρ_ζ	AR preference	0.10	0.17
ρ_{z^I}	AR stationary investment-specific productivity	0.21	0.47
σ_z^0	std. stationary neutral productivity 0	0.62	0.65
σ_z^4	std. stationary neutral productivity 4	0.11	0.11
σ_z^8	std. stationary neutral productivity 8	0.11	0.09
$\sigma_{\mu^a}^0$	std. non-stationary investment-specific productivity 0	0.16	0.21
$\sigma_{\mu^a}^4$	std. non-stationary investment-specific productivity 4	0.20	0.16
$\sigma_{\mu^a}^8$	std. non-stationary investment-specific productivity 8	0.19	0.16
σ_g^0	std. government spending 0	0.53	0.62
σ_g^4	std. government spending 4	0.69	0.57
σ_g^8	std. government spending 8	0.43	0.37
$\sigma_{\mu^x}^0$	std. non-stationary neutral productivity 0	0.45	0.38
$\sigma_{\mu^x}^4$	std. non-stationary neutral productivity 4	0.12	0.08
$\sigma_{\mu^x}^8$	std. non-stationary neutral productivity 8	0.12	0.10
σ_μ^0	std. wage markup 0	1.51	0.50
σ_μ^4	std. wage markup 4	3.93	4.79
σ_μ^8	std. wage markup 8	3.20	0.51
σ_ζ^0	std. preference 0	2.83	4.03
σ_ζ^4	std. preference 4	2.76	1.89
σ_ζ^8	std. preference 8	5.34	2.21
$\sigma_{z^I}^0$	std. stationary investment-specific productivity 0	34.81	11.72
$\sigma_{z^I}^4$	std. stationary investment-specific productivity 4	11.99	1.93
$\sigma_{z^I}^8$	std. stationary investment-specific productivity 8	14.91	5.50

Note: The values are taken from Table II of Schmitt-Grohé and Uribe (2012)

Table B2: CRLBs

	parameter	$\bar{\boldsymbol{y}}$	$\boldsymbol{y}$
θ	Frisch elasticity of labor supply	1.65135	0.27551
γ	wealth elasticity of labor supply	0.00002	0.00001
κ	investment adjustment cost	30.63677	1.50300
δ_2/δ_1	capacity utilization cost	0.03112	0.00040
b	habit in consumption	0.00018	0.00004
ρ_{xg}	government spending	0.04158	0.02751
ρ_z	AR stationary neutral productivity	0.00178	0.00026
ρ_{μ^a}	AR non-stationary investment-specific productivity	0.00371	0.00222
ρ_g	AR government spending	0.00116	0.00055
ρ_{μ^x}	AR non-stationary neutral productivity	0.15107	0.02009
ρ_μ	AR wage markup	0.00045	0.00011
ρ_ζ	AR preference	0.00685	0.00491
ρ_{z^I}	AR stationary investment-specific productivity	0.02235	0.00100
σ_z^0	std. stationary neutral productivity	0.03879	0.00488
σ_z^4	std. stationary neutral productivity q4	2.09192	0.15462
σ_z^8	std. stationary neutral productivity q8	1.51431	0.14918
$\sigma_{\mu^a}^0$	std. non-stationary investment-specific productivity	0.03482	0.00177
$\sigma_{\mu^a}^4$	std. non-stationary investment-specific productivity q4	0.04562	0.00156
$\sigma_{\mu^a}^8$	std. non-stationary investment-specific productivity q8	0.04740	0.00167
σ_g^0	std. government spending	0.61389	0.02131
σ_g^4	std. government spending q4	1.25121	0.11185
σ_g^8	std. government spending q8	3.22258	0.27854
$\sigma_{\mu^x}^0$	std. non-stationary neutral productivity	0.09638	0.02098
$\sigma_{\mu^x}^4$	std. non-stationary neutral productivity q4	1.43272	0.08717
$\sigma_{\mu^x}^8$	std. non-stationary neutral productivity q8	0.75869	0.07645
σ_μ^0	std. wage markup	6.52819	0.07706
σ_μ^4	std. wage markup q4	4.78828	0.46297
σ_μ^8	std. wage markup q8	6.02596	0.59652
σ_ζ^0	std. preference	59.14528	0.98231
σ_ζ^4	std. preference q4	99.49705	9.82559
σ_ζ^8	std. preference q8	33.00313	3.02169
$\sigma_{z^I}^0$	std. stationary investment-specific productivity	70.24558	5.52009
$\sigma_{z^I}^4$	std. stationary investment-specific productivity q4	104.96769	4.48185
$\sigma_{z^I}^8$	std. stationary investment-specific productivity q8	44.03072	3.50932

Note: $\boldsymbol{y}$ includes all observables, $\bar{\boldsymbol{y}} = \boldsymbol{y} \setminus (v^f, r)$. The bounds are computed for the MLE values in Table B1 with $T = 207$.

Table B3: Efficiency gains (%)

	parameter	v^f, r	v^f	r
θ	Frisch elasticity of labor supply	83	61	78
γ	wealth elasticity of labor supply	51	35	28
κ	investment adjustment cost	95	94	59
δ_2/δ_1	capacity utilization cost	99	97	88
b	habit in consumption	75	66	27
ρ_{xg}	government spending	34	17	20
ρ_z	AR stationary neutral productivity	85	66	67
ρ_{μ^a}	AR non-stationary investment-specific productivity	40	37	2
ρ_g	AR government spending	52	48	42
ρ_{μ^x}	AR non-stationary neutral productivity	87	72	60
ρ_μ	AR wage markup	77	69	14
ρ_ζ	AR preference	28	19	9
ρ_{z^I}	AR stationary investment-specific productivity	96	92	78
σ_z^0	std. stationary neutral productivity	87	59	83
σ_z^4	std. stationary neutral productivity q4	93	73	72
σ_z^8	std. stationary neutral productivity q8	90	73	64
$\sigma_{\mu^a}^0$	std. non-stationary investment-specific productivity	95	95	43
$\sigma_{\mu^a}^4$	std. non-stationary investment-specific productivity q4	97	96	74
$\sigma_{\mu^a}^8$	std. non-stationary investment-specific productivity q8	96	96	68
σ_g^0	std. government spending	97	80	95
σ_g^4	std. government spending q4	91	89	52
σ_g^8	std. government spending q8	91	89	55
$\sigma_{\mu^x}^0$	std. non-stationary neutral productivity	78	50	67
$\sigma_{\mu^x}^4$	std. non-stationary neutral productivity q4	94	78	71
$\sigma_{\mu^x}^8$	std. non-stationary neutral productivity q8	90	74	58
σ_μ^0	std. wage markup	99	70	97
σ_μ^4	std. wage markup q4	90	84	52
σ_μ^8	std. wage markup q8	90	85	48
σ_ζ^0	std. preference	98	89	97
σ_ζ^4	std. preference q4	90	88	39
σ_ζ^8	std. preference q8	91	88	50
$\sigma_{z^I}^0$	std. stationary investment-specific productivity	92	91	66
$\sigma_{z^I}^4$	std. stationary investment-specific productivity q4	96	93	81
$\sigma_{z^I}^8$	std. stationary investment-specific productivity q8	92	88	72

Note: The efficiency gain $\text{EG}_{\theta_i}(\boldsymbol{x}|\bar{\boldsymbol{y}})$, for (1) $\boldsymbol{x} = (v^f, r)$, (2) $\boldsymbol{x} = v^f$, or (3) $\boldsymbol{x} = r$, is defined as the reduction in the value of CRLB for θ_i when all variables are observed, as a per cent of the value of the CRLB when all variables except those in $\boldsymbol{x}$ are observed.

Table B4: Information content of asset prices: different estimates of SGU model

		A. SGU median		B. HS(2014)		C. MN(2015)	
	innovation	IG($\bar{\boldsymbol{y}}$)	IG($v^f, r\|\bar{\boldsymbol{y}}$)	IG($\bar{\boldsymbol{y}}$)	IG($v^f, r\|\bar{\boldsymbol{y}}$)	IG($\bar{\boldsymbol{y}}$)	IG($v^f, r\|\bar{\boldsymbol{y}}$)
$\varepsilon^0_{\mu^a}$	non-stationary investment-specific prod.	47.6	8.8	46.5	1.8	92.7	0.0
$\varepsilon^4_{\mu^a}$	non-stationary investment-specific prod. 4q	25.8	10.7	26.6	2.0	3.1	0.1
$\varepsilon^8_{\mu^a}$	non-stationary investment-specific prod. 8q	26.0	9.1	26.1	1.9	4.1	0.1
$\varepsilon^0_{\mu^x}$	non-stationary neutral prod.	24.6	39.9	41.2	45.9	47.4	43.6
$\varepsilon^4_{\mu^x}$	non-stationary neutral prod. 4q	1.3	8.2	3.1	9.9	3.3	4.7
$\varepsilon^8_{\mu^x}$	non-stationary neutral prod. 8q	1.9	11.1	5.5	13.2	7.9	7.0
$\varepsilon^0_{z^I}$	stationary investment-specific prod.	89.1	5.7	81.4	10.4	73.3	12.9
$\varepsilon^4_{z^I}$	stationary investment-specific prod. 4q	1.7	3.8	4.0	4.8	16.2	9.0
$\varepsilon^8_{z^I}$	stationary investment-specific prod. 8q	14.6	30.1	12.8	15.7	4.8	3.0
ε^0_{z}	stationary neutral prod.	80.2	9.0	81.5	10.9	33.3	29.4
ε^4_{z}	stationary neutral prod. 4q	2.5	17.6	3.5	14.5	10.4	3.5
ε^8_{z}	stationary neutral prod. 8q	1.8	12.9	2.9	12.2	43.0	14.6
ε^0_{μ}	wage markup	1.4	7.2	5.6	21.1	52.1	20.1
ε^4_{μ}	wage markup 4q	96.4	1.2	84.5	3.0	1.0	0.5
ε^8_{μ}	wage markup 8q	1.1	0.3	8.5	2.1	47.7	21.9
ε^0_{g}	government spending	43.5	3.2	42.7	2.4	93.9	2.7
ε^4_{g}	government spending 4q	37.1	3.5	33.5	2.3	0.5	0.1
ε^8_{g}	government spending 8q	15.2	1.6	18.3	1.2	0.6	0.1
ε^0_{ζ}	preference	61.2	7.2	42.2	9.9	79.5	19.5
ε^4_{ζ}	preference 4q	13.9	3.4	17.6	3.4	0.2	0.1
ε^8_{ζ}	preference 8q	18.8	4.5	20.7	3.9	0.2	0.0

Note: See the note to Table 1 in the main text. The information gains are evaluated at three point estimated of the SGU model: (1) the posterior median in SGU, (2) the posterior mean in Herbst and Schorfheide (2014), and (3) the posterior median in Miyamoto and Nguyen (2015).

	y	c	i	h	g	a	tfp
$\varepsilon^0_{\mu^a}$	0	1.8	0.81	13	1.4	-0.13	29
$\varepsilon^4_{\mu^a}$	0	1.7	0.64	14	1.2	-0.1	41
$\varepsilon^8_{\mu^a}$	0	1.9	0.74	14	1.4	-0.07	33
$\varepsilon^0_{\mu^x}$	0	3.2	1.5	7.2	1.1	0.58	0.09
$\varepsilon^4_{\mu^x}$	0	1.3	0.63	3.9	1.1	0.1	1.1
$\varepsilon^8_{\mu^x}$	0	1	0.6	3	1.2	0.14	0.63
$\varepsilon^0_{z^I}$	0	1.3	-0.16	3.2	1.3	-0.05	0.35
$\varepsilon^4_{z^I}$	0	1.5	0.07	6.6	1.1	0.09	5.8
$\varepsilon^8_{z^I}$	0	1.2	0.08	4.5	1.1	0.16	4.2
ε^0_{z}	0	3.4	1.4	4.9	0.65	0.7	-0.45
ε^4_{z}	0	1.3	0.72	3.3	0.94	0.1	0.27
ε^8_{z}	0	1.2	0.82	3.1	1.1	0.2	0.2
ε^0_{μ}	0	0.03	0.05	-0.44	0.1	0.04	-0.21
ε^4_{μ}	0	-0.22	-0.13	-0.46	-0.12	0.01	-0.44
ε^8_{μ}	0	-0.06	0.04	-0.44	0.1	0.09	-0.39
ε^0_{g}	0	1.1	0.51	2.6	-0.38	0.02	0.43
ε^4_{g}	0	0.88	0.39	1.5	-0.31	0.02	0.56
ε^8_{g}	0	1	0.48	1.6	-0.3	0.11	0.74
ε^0_{ζ}	0	-0.39	0.29	-0.1	0.31	0.04	-0.16
ε^4_{ζ}	0	-0.35	0.24	-0.18	0.24	0	-0.26
ε^8_{ζ}	0	-0.37	0.18	-0.35	0.06	-0.02	-0.41

Figure B1: Conditional pairwise complementarity between v^f and macro variables at MLE in Schmitt-Grohé and Uribe (2012)

	y	c	i	h	g	a	tfp
$\varepsilon^{0}_{\mu^a}$	0	0.55	0.28	0.79	0.41	-0.01	1.1
$\varepsilon^{4}_{\mu^a}$	0	0.93	0.49	3.2	0.42	-0.02	3.2
$\varepsilon^{8}_{\mu^a}$	0	0.78	0.41	2.2	0.42	-0.01	2.4
$\varepsilon^{0}_{\mu^x}$	0	0.3	0.13	1.6	0.27	0	-0.05
$\varepsilon^{4}_{\mu^x}$	0	0.38	0.19	2.5	0.28	0.02	0.13
$\varepsilon^{8}_{\mu^x}$	0	0.33	0.17	2	0.26	0.01	0.12
$\varepsilon^{0}_{z^I}$	0	0.69	-0.02	2.2	0.43	-0.04	0.5
$\varepsilon^{4}_{z^I}$	0	0.66	-0.03	2.9	0.3	0.03	1.4
$\varepsilon^{8}_{z^I}$	0	0.53	-0.07	1.9	0.29	0.04	1.1
ε^{0}_{z}	0	0.37	0.18	1.1	0.15	0	-0.3
ε^{4}_{z}	0	0.45	0.2	3.3	0.35	0.02	0.18
ε^{8}_{z}	0	0.43	0.19	2.9	0.34	0.02	0.17
ε^{0}_{μ}	0	0.8	0.33	2.1	0.49	0.02	3.1
ε^{4}_{μ}	0	-0.3	-0.24	-0.46	-0.22	-0.01	-0.43
ε^{8}_{μ}	0	-0.29	-0.23	-0.46	-0.21	-0.01	-0.42
ε^{0}_{g}	0	0.42	0.19	3	0.05	0.01	0.77
ε^{4}_{g}	0	0.28	0.17	1.3	-0.25	0.06	0.94
ε^{8}_{g}	0	0.17	0.12	0.8	-0.25	0.06	0.64
ε^{0}_{ζ}	0	0.3	0.15	1.5	0.35	0.02	0.88
ε^{4}_{ζ}	0	-0.29	-0.11	-0.26	0.04	0	-0.26
ε^{8}_{ζ}	0	-0.29	-0.1	-0.27	0.02	0	-0.28

Figure B2: Conditional pairwise complementarity between r and macro variables at MLE in Schmitt-Grohé and Uribe (2012)

	y	c	i	h	g	a	tfp
$\varepsilon^0_{\mu^a}$	0.1	0.1	0.2	0	-0.1	0	1.7
$\varepsilon^4_{\mu^a}$	0.1	0.2	0.1	0	0	0	1.6
$\varepsilon^8_{\mu^a}$	0.1	0.2	0.1	0	0	0	1.6
$\varepsilon^0_{\mu^x}$	-0.1	-0.1	-0.1	0	-0.1	0	-0.4
$\varepsilon^4_{\mu^x}$	-0.1	0	0	0	-0.1	0	-0.1
$\varepsilon^8_{\mu^x}$	0	0.1	0	0	-0.1	0	-0.1
$\varepsilon^0_{z^I}$	0.1	0.1	0	0.1	0.1	0	1.3
$\varepsilon^4_{z^I}$	0.3	0.5	0.1	0	0	0	1.7
$\varepsilon^8_{z^I}$	0.5	0.6	0.2	0.1	0	0	1.6
ε^0_{z}	-0.1	0	-0.1	0	0	0	-0.4
ε^4_{z}	0	0	0	0	0	0	-0.2
ε^8_{z}	0	0.1	0	0	0	0	-0.2
ε^0_{μ}	-0.1	-0.1	-0.1	0	0	0	1.7
ε^4_{μ}	0	-0.1	0	0	0	0	1.7
ε^8_{μ}	0	0	0	0	0	0	1.7
ε^0_{g}	0	-0.1	-0.1	0	0	0	1.7
ε^4_{g}	0	0	0	0	0	0	1.6
ε^8_{g}	0	0.1	0	0	0	0	1.7
ε^0_{ζ}	0	0.1	-0.1	0	0	0	1.7
ε^4_{ζ}	0	0	0	0	0	0	1.7
ε^8_{ζ}	0	0	0	0	0	0	1.7

Figure B3: Unconditional pairwise complementarity between v^f and macro variables at MLE in Schmitt-Grohé and Uribe (2012)

	y	c	i	h	g	a	tfp
$\varepsilon^0_{\mu^a}$	-0.05	0.06	-0.01	0.26	-0.15	0	0.74
$\varepsilon^4_{\mu^a}$	-0.1	0.04	-0.02	0.67	-0.06	0	0.23
$\varepsilon^8_{\mu^a}$	-0.1	0.06	-0.02	0.62	-0.05	0	0.22
$\varepsilon^0_{\mu^x}$	-0.12	-0.08	-0.09	0.25	-0.09	0.01	-0.03
$\varepsilon^4_{\mu^x}$	0.06	0.05	0.04	0.72	0.01	0	-0.03
$\varepsilon^8_{\mu^x}$	0.08	0.07	0.05	0.71	0.01	0	0
$\varepsilon^0_{z^I}$	0.06	0.28	0	0.42	0.04	0	0.43
$\varepsilon^4_{z^I}$	-0.1	0.15	-0.06	0.51	0.03	0	0.22
$\varepsilon^8_{z^I}$	-0.1	0.23	-0.07	0.44	0.03	0	0.36
ε^0_{z}	-0.13	-0.07	-0.09	0.29	0.09	0.01	-0.09
ε^4_{z}	0.05	0.03	0.03	0.8	0.02	0	-0.07
ε^8_{z}	0.06	0.04	0.03	0.79	0.02	0	-0.06
ε^0_{μ}	-0.18	-0.13	-0.1	0.17	0.05	0.01	0.62
ε^4_{μ}	0.04	0.01	0.01	-0.29	0.02	0	0.05
ε^8_{μ}	0.06	0.03	0.03	-0.28	0.02	0	0.06
ε^0_{g}	0.02	-0.12	-0.09	0.35	0.02	0.01	0.6
ε^4_{g}	-0.1	0.03	0.03	0.86	-0.03	0	0.06
ε^8_{g}	-0.11	0.05	0.03	0.85	-0.03	0	0.06
ε^0_{ζ}	0.02	0.03	-0.06	0.28	0.05	0.01	0.6
ε^4_{ζ}	-0.1	-0.04	0.04	0.82	0.02	0	0.07
ε^8_{ζ}	-0.1	-0.04	0.05	0.83	0.02	0	0.08

Figure B4: Unconditional pairwise complementarity between r and macro variables at MLE in Schmitt-Grohé and Uribe (2012)

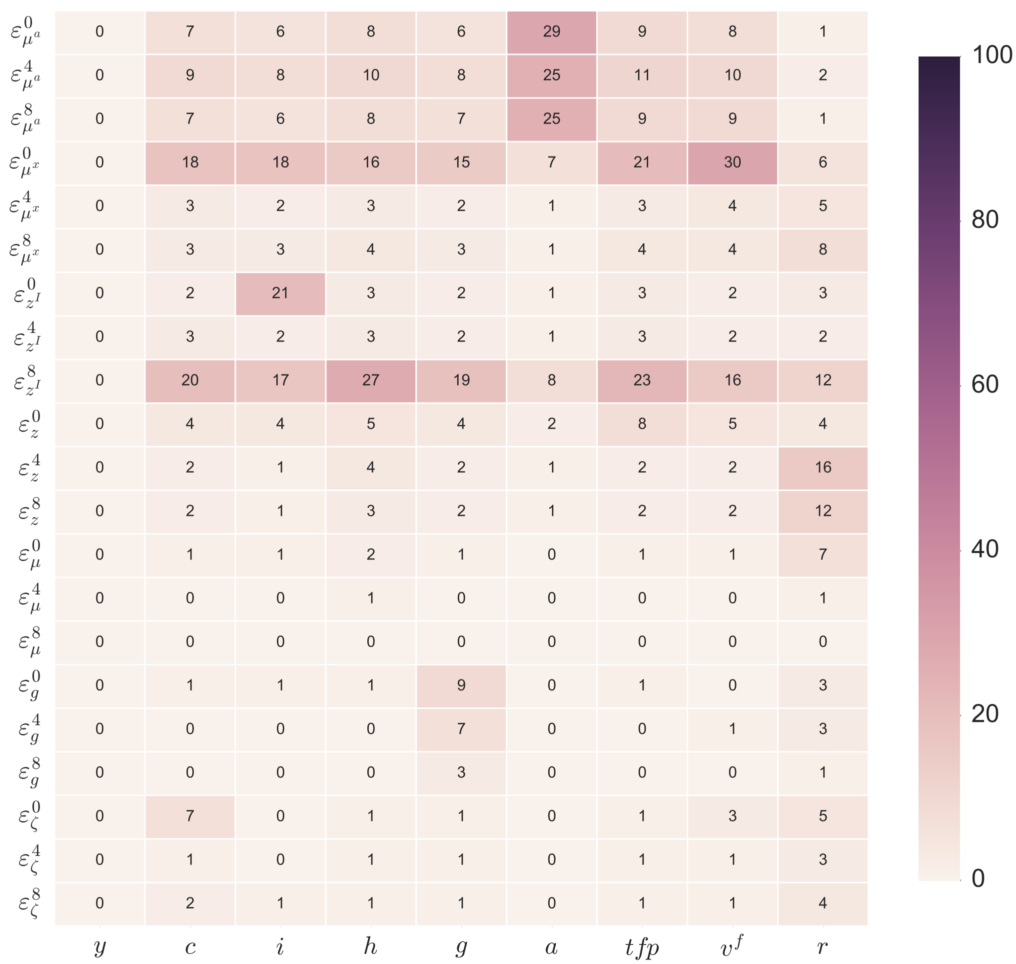

Figure B5: Conditional information gains at the posterior median in Schmitt-Grohé and Uribe (2012)

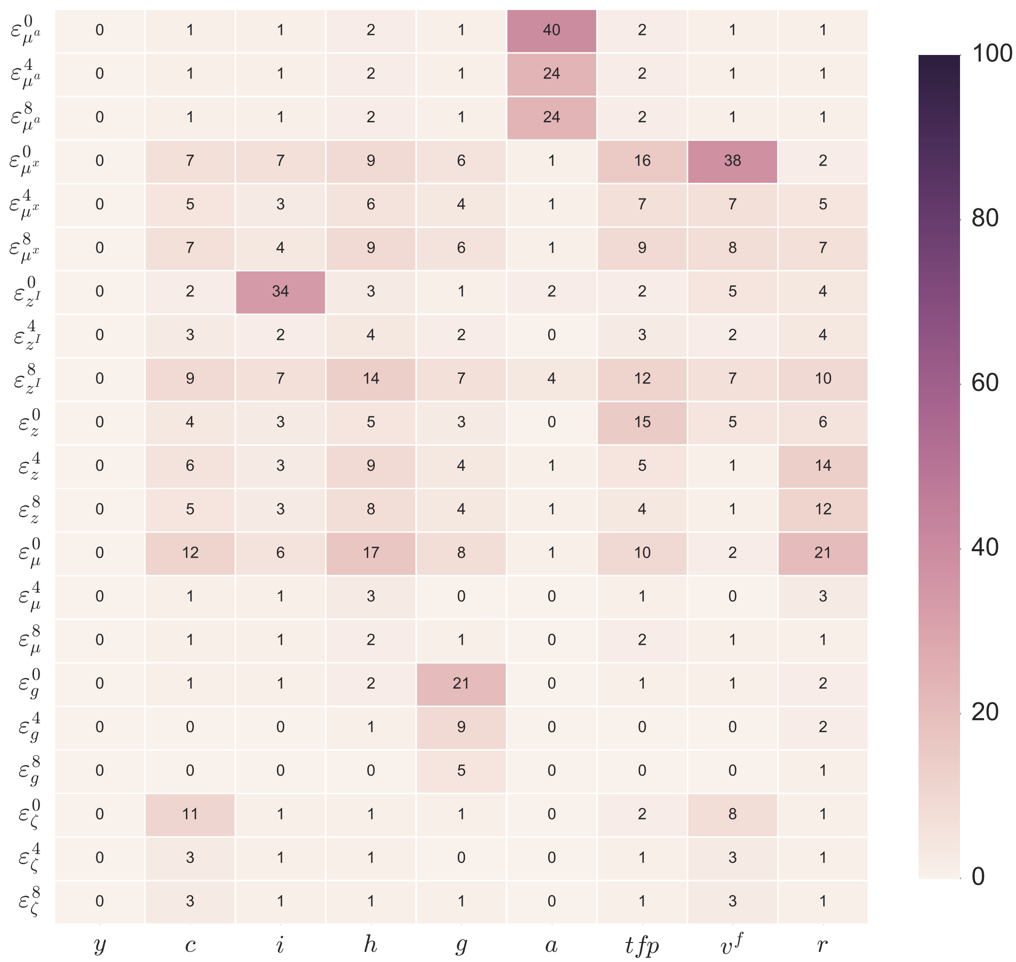

Figure B6: Conditional information gains at the posterior mean in Herbst and Schorfheide (2014)

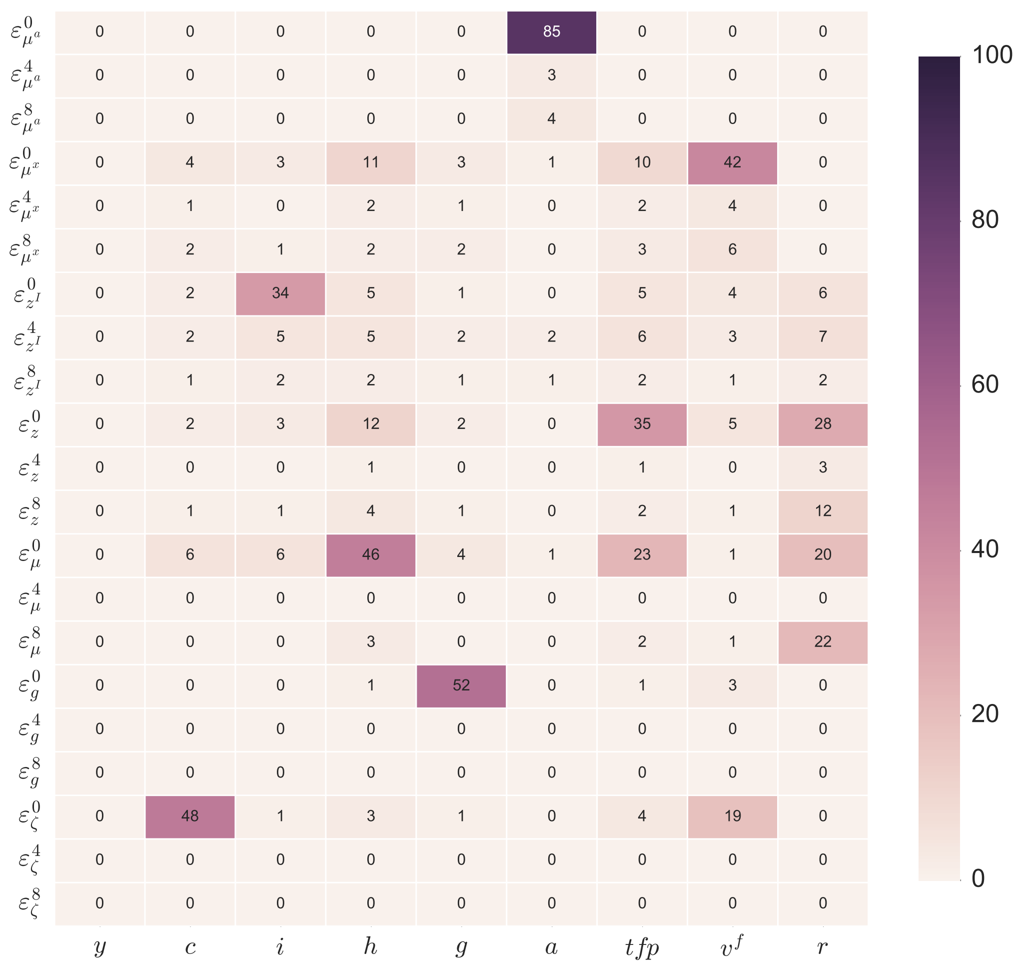

Figure B7: Conditional information gains at the posterior median in Miyamoto and Nguyen (2015)

C Avdjiev (2016) model

The representative agent maximizes the following utility function:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(C_t - \theta_c C_{t-1})\,(l_t - \theta_l l_{t-1})^{\chi}\right]^{1-\gamma} - 1}{1-\gamma}, \tag{C.1}$$

where C_t is consumption, l_t is leisure, β is the discount factor, γ is the inverse of the intertemporal elasticity of substitution, χ determines the Frisch elasticity of labor supply, θ_c and θ_l are parameters determining the degrees of habit persistence in consumption and leisure, respectively. Output is produced using:

$$Y_t = Z_t (u_t K_t)^{\alpha} (X_t h_t)^{1-\alpha}, \tag{C.2}$$

where K_t is the existing capital stock, $h_t = 1 - l_t$ is hours worked, u_t is the rate of capacity utilization, Z_t is a stationary neutral productivity shock, and X_t is a non-stationary neutral productivity shock.
The law of motion for the stock of capital is:

$$K_{t+1} = (1 - \delta(u_t))K_t + \Omega_t \left[I_t - \frac{1}{2\delta_0 \eta} \left(\frac{I_t}{K_t} - \tau \right)^2 K_t \right], \tag{C.3}$$

where I_t is investment, δ is the rate of depreciation and is an increasing function of the rate of capacity utilization, Ω_t is a stationary investment-specific productivity shock, τ is the steady-state level of the investment-capital ratio, η is the elasticity of the investment-capital ratio with respect to Tobin's q, and δ_0 is the steady-state capital depreciation rate.

There is no government in this economy and output is used for either consumption or investment:

$$Y_t = C_t + I_t A_t, \tag{C.4}$$

where A_t is a non-stationary investment specific productivity shock.

The main departure from the SGU model is in the way news shocks are introduced into the model. In particular, the specification of shock precesses in (B.6) is replaced with

$$\begin{aligned} \ln(x_t/x) &= \rho_x^l \ln(x_{t-1}/x) + (1 - \rho_x^l) \ln(x_{t-1}^{LR}) + \sigma_{x,u} \varepsilon_{x,t}^u \\ \ln(x_t^{LR}) &= \rho_x^{LR} \ln(x_{t-1}^{LR}) + \sigma_{x,LR} \varepsilon_{x,t}^{LR}, \end{aligned} \tag{C.5}$$

where ε_x^u and ε_x^{LR} are independent standard normal random variables. Avdjiev (2016) further assumes that $0 < \rho_x^l < 1$ and $\rho_x^{LR} = 0.999$, which implies that $\ln(x_t^{LR})$ can be interpreted as the long-run component of $\ln(x_t/x)$.[3] Therefore, ε_x^{LR} is the anticipated change in the long-run value of the shock. The model contains only four of the seven

[3]Note that if $\rho_x^{LR} = 1$ then $\lim_{s\to\infty} \mathrm{E}_t \ln(x_{t+s}/x) = \ln(x^{LR})$.

Table C1: Information content of asset prices: innovations

	innovation	IG($\bar{\boldsymbol{y}}$)	IG($v^f, r\|\bar{\boldsymbol{y}}$)	IG($v^f\|\bar{\boldsymbol{y}}$)	IG($r\|\bar{\boldsymbol{y}}$)	IG(v^f)	IG(r)
ε_x^u	non-stat. neutral prod.	92.4	5.4	0.1	5.4	3.3	0.3
ε_x^{LR}	non-stat. neutral prod. LR news	0.2	0.2	0.1	0.1	0.1	0.0
ε_a^u	non-stat. investment-specific prod.	99.5	0.4	0.3	0.0	1.7	0.8
ε_a^{LR}	non-stat. investment-specific prod. LR news	42.8	44.7	36.9	3.6	60.5	1.8
ε_z^u	stat. neutral prod.	94.8	4.0	0.4	3.8	1.8	52.7
ε_z^{LR}	stat. neutral prod. LR news	37.2	48.0	9.5	43.0	12.4	31.8
ε_ω^u	stat. investment-specific prod.	57.8	35.4	35.3	1.1	7.5	3.7
ε_ω^{LR}	stat. investment-specific prod. LR news	0.0	0.0	0.0	0.0	0.0	0.0

Note: see the note to Table 1 in the main text. $\bar{\boldsymbol{y}}$ includes the growth rates of output, consumption, and investment, hours worked, and the relative price of investment.

fundamental shocks present in the SGU model, namely: stationary and non-stationary neutral productivity shocks and stationary and non-stationary investment-specific productivity shocks. All shocks evolve as in (C.5), implying that there are four different long-run components and eight exogenous innovations, four of which are interpreted as long run (LR) news shocks.

Avdjiev (2016) argues that the long-run specification of news shocks fits the data better than the specification in (B.6). Importantly, Avdjiev (2016) uses asset price data to estimate the model. The two asset price variables used are the growth rate of the total stock market valuation and the real risk-free rate. These variables are assumed to be noisy measures of v^f and r, which are defined as in Section 4. In addition, five macroeconomic variables are used in the estimation: the growth rates of output (y_t), consumption (c_t), and investment (i_t), hours worked (h_t), and the relative price of investment (a_t).

C.1 Information about news shocks

I proceed along the lines of the analysis carried out in Section Section 4.1. Note that now $\boldsymbol{y}$ is a $T \times 7$ dimensional vector, and $\bar{\boldsymbol{y}} = \boldsymbol{y} \backslash (v^f, r)$ is a $T \times 5$ dimensional vector. I set $T = 236$, which is the sample size in Avdjiev (2016), and assume that $\boldsymbol{\theta}$ is equal to the median of the posterior distribution reported in Avdjiev (2016) (see Table C5).

The results are presented in Table C1. Including asset prices among the observables leads to significant information gains with respect to two of the news shocks – the anticipated innovations in the non-stationary investment-specific productivity shock ε_a^{LR}, and the stationary neutral productivity shock ε_z^{LR}. Almost all of the gains in the first case are due to including v^f, while in the second most of the information is contributed by r. However, neither one of the innovations can be fully recovered from $\boldsymbol{y}$. The total information gains are around 88% for ε_a^{LR} and 85% for ε_z^{LR}. The respective unconditional gains reported in the last two columns are relatively large, implying that, in contrast to the SGU model, v^f and r contribute significant amounts of non-redundant information with respect to ε_a^{LR} and ε_z^{LR}. There is essentially no information in the set of observables as a whole about the news components of the other two shocks – the non-stationary neutral productivity shock and the stationary investment-specific productivity shock. This can be understood from the observation that the standard deviations of these innovations are estimated to be very small compared to the standard deviations of the unanticipated innovations to the same shocks.[4] Furthermore, since the stationary investment-specific productivity shock is estimated to be very persistent, the coefficient on the long-run component in (C.5) is close to zero, making ε_ω^{LR} very difficult to identify.

In addition to the two news shocks components, asset prices, and in particular v^f, contribute a significant amount of information with respect to the unanticipated innovations to the stationary investment specific productivity shock ε_ω^u. The small size of the unconditional gain implies that the contribution of v^f is largely a result of the interactions of that variable with variables in $\bar{\boldsymbol{y}}$.

To find out how the contributions of v^f and r compare to other variable, Figure C1 presents conditional information gains for each one of the seven observables. The results show that v^f and r are indeed the most informative variables with respect to the two identified news shocks. In the case of ε_z^{LR}, the conditional information gain from observing hours worked is somewhat larger than the gain from including v^f, but is smaller than the information gain from observing r. The relative price of investment is the only other observable with a positive marginal contribution. None of the macro variables makes a positive contribution with respect to ε_a^{LR}. Note that, as in Section 4, the gains from each variable are conditional on information contained in the remaining six variables. This is why the results for v^f and r are different from the values in Table C1 (columns 3 and 4). In particular, the information gains with respect to ε_a^{LR} due to either v^f or r are larger when the conditioning set includes the other asset price variable compared to when it does not. This indicates a positive conditional information complementarity between v^f and r with respect to that shock. At the same time, there is a negative complementarity with respect to the news component in the stationary neutral productivity shock.[5] Additional results from conditional and unconditional information complementarity analysis are presented in Figures C4 – C7. There is a significant information complementarity between v^f and a with respect to the news components in two of the shocks – negative with respect to the stationary neutral productivity shock, and positive with respect to the stationary investment-specific productivity shock. There is also a positive complementarity between v^f and h with respect to the stationary neutral productivity shock. In the case of r, the only significant complementarity is with a – positive with respect to the news component in the non-stationary neutral productivity shock.

Another interesting result in Figure C1 is the apparent lack of information in y, i

[4]The posterior median estimates are: $\sigma_x^{LR} = 0.01$ vs. $\sigma_x^u = 1.05$ and $\sigma_\omega^{LR} = 0.07$ vs. $\sigma_\omega^u = 9.97$.

[5]Another way to see this is by comparing the joint information gains to the sum of the individual gains in Table C1.

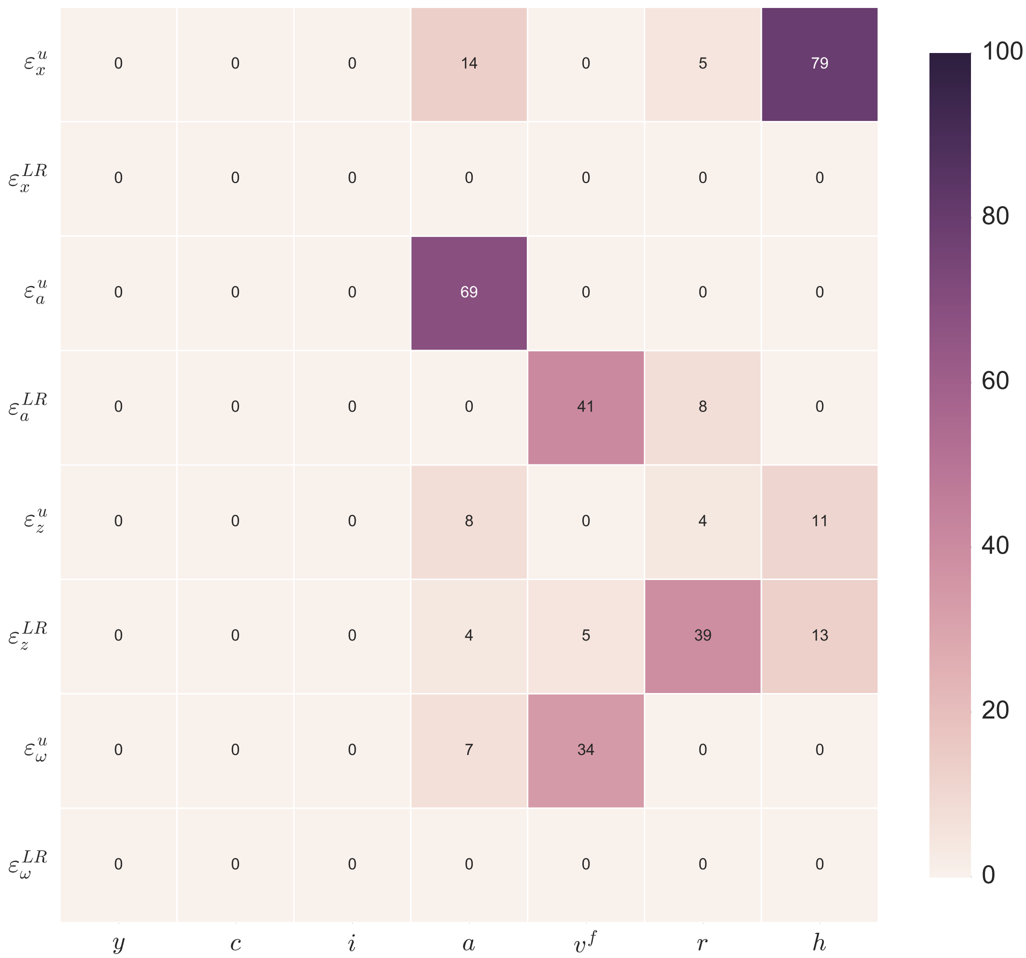

Figure C1: Conditional information gains in the model of Avdjiev (2016).

and c. In fact, the conditional information gains are positive but very small, suggesting near redundancy of these variables. This is easily explained by the observation that the economy's resource constraint (see equation (C.4)) implies linear dependence among y, i and c.[6] Stochastic singularity is avoided by assuming measurement errors in all variables. However, the size of the errors in y, i and c is very small, implying that any one of them is (nearly) redundant given the other two.

Table C2 reports results on the information content of asset prices with respect to the four structural shocks and their long-run components. The information gains are very similar to the ones with respect to the innovations, presented in Table C1, both in terms of the size of the gains and the contribution of each asset prices variable. This is

[6]As in the SGU model, the observed investment series is defined as $i_t := \triangle \ln A_t I_t$.

Table C2: Information content of asset prices: shocks

	shock	IG($\bar{\boldsymbol{y}}$)	IG($v^f, r\|\bar{\boldsymbol{y}}$)	IG($v^f\|\bar{\boldsymbol{y}}$)	IG($r\|\bar{\boldsymbol{y}}$)	IG(v^f)	IG(r)
μ_x	non-stat. neutral prod.	93.3	4.9	0.1	4.9	3.3	0.4
μ_x^{LR}	non-stat. neutral prod. LR comp.	0.8	0.4	0.1	0.3	0.1	0.0
μ_a	non-stat. investment-specific prod.	100.0	0.0	0.0	0.0	5.2	3.5
μ_a^{LR}	non-stat. investment-specific prod. LR comp.	44.3	43.5	35.9	3.5	60.7	3.3
z	stat. neutral prod.	95.9	3.0	0.8	2.5	4.6	63.1
z^{LR}	stat. neutral prod. LR comp.	37.8	47.5	9.4	42.5	12.4	31.7
ω	stat. investment-specific prod.	57.7	35.3	35.3	1.1	7.5	3.7
ω^{LR}	stat. investment-specific prod. LR comp.	0.0	0.0	0.0	0.0	0.0	0.0

Note: see note to Table 1 in the main text.

to be expected given that shocks and innovations are closely linked to each other in this model.

C.2 Information about parameters

Table C3 reports parameter efficiency gains due to observing the two asset price variables. The gains with respect to the standard deviations of the four news shocks are between 25% and 86%. Similar to the information gains results in Table C1, v^f is relatively more informative for the parameters of the two investment-specific productivity shocks, while r is more informative about the parameters of the stationary and non-stationary neutral productivity shocks. Notice that this applies to all parameters of the same shock, including the autoregressive coefficients and the standard deviations of the unanticipated shocks.

It is worth pointing out that the news shock parameters, and in particular $\sigma_{x,LR}$ and $\sigma_{\omega,LR}$, are identified, in spite of the earlier finding that there is very little information about the realizations of the news components of the non-stationary neutral productivity and stationary investment-specific productivity shocks. Lack of identification would imply an infinite value of the CRLB. As can be seen in Table C6, which shows the CRLBs with and without asset prices, they are all finite. The values for $\sigma_{\omega,LR}$, however, are very large, suggesting that the likelihood surface is in fact very flat with respect to that parameter. Two other parameters with extremely large values of the CRLB are the cost of capacity utilization parameter δ_2 and the Frisch elasticity of labor supply χ. Notice that, even though the values of the CRLBs of $\sigma_{\omega,LR}$ an δ_2 are almost equal, the levels of uncertainty they imply are very different since $\delta_2 = 3.91$ while $\sigma_{\omega,LR} = 0.07$.

To find out how v^f and r compare to other observables, Figure C2 shows the efficiency gains due to each one of the seven variables. Only with respect to one of the news shock parameters – the standard deviation of the long-run news component in the non-stationary investment specific-productivity shock ($\sigma_{a,LR}$), is v^f significantly more informative than any other variable. The relative price of investment is about as

Table C3: Efficiency gains (%)

	parameter	v^f, r	v^f	r
γ	inverse intertemporal elasticity of substitution	18	1	16
χ	Frisch elasticity of labor supply	99	44	98
θ_l	habit in leisure,	40	14	37
θ_c	habit in consumption,	36	10	26
δ_2	capacity utilization cost	88	83	26
η	investment adjustment cost	88	85	21
ρ_x^l	AR non-stationary neutral productivity	85	1	85
ρ_a^l	AR non-stationary investment-specific productivity	19	13	10
ρ_z^l	AR stationary neutral productivity	90	17	88
ρ_ω^l	AR stationary investment-specific productivity	52	47	15
$\sigma_{x,u}$	std. non-stationary neutral productivity	33	5	31
$\sigma_{x,LR}$	std. non-stationary neutral productivity LR news	25	1	25
$\sigma_{a,u}$	std. non-stationary investment-specific productivity	5	4	1
$\sigma_{a,LR}$	std. non-stationary investment-specific productivity LR news	69	54	17
$\sigma_{z,u}$	std. stationary neutral productivity	45	13	45
$\sigma_{z,LR}$	std. stationary neutral productivity LR news	64	24	50
$\sigma_{\omega,u}$	std. stationary investment-specific productivity	90	89	18
$\sigma_{\omega,LR}$	std. stationary investment-specific productivity LR news	86	86	10

Note: see note to Table 3 in the main text.

informative as v^f with resect to the standard deviation of the long-run news component in the stationary investment-specific productivity shock ($\sigma_{\omega,LR}$), and is also by far the most informative variable with respect to the standard deviation of the long-run news component in the non-stationary neutral productivity shock ($\sigma_{x,LR}$). Lastly, hours worked is the most informative variable with respect to the standard deviation of the long-run news component in the stationary neutral productivity shock ($\sigma_{z,LR}$).

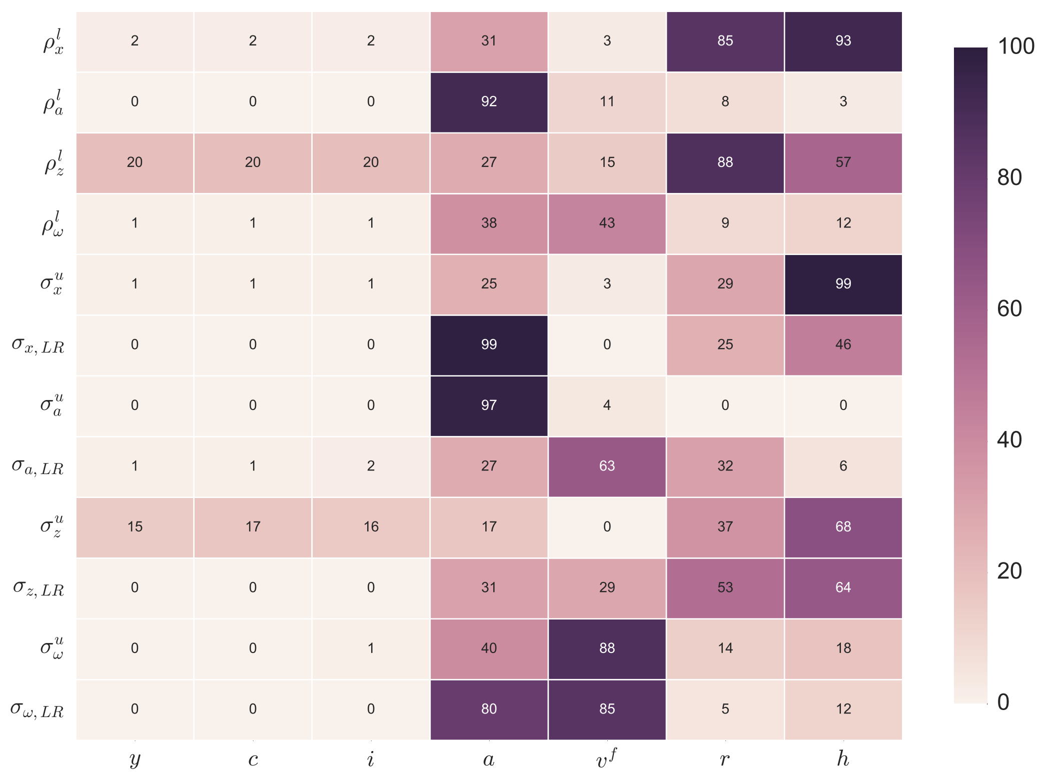

Figure C2: Efficiency gains in the model of Avdjiev (2016).

It should be noted that these results are obtained under the assumption that the standard deviations of the measurement errors are known. Without it the efficiency gains cannot be computed since the measurement error parameters are not identified unless the respective variables are observed. This does not affect the conclusions regarding the contributions of asset prices, but does inflate the efficiency gains with respect to ρ_z^l and $\sigma_{z,u}$ due to y, c, and i.[7]

[7]There are also relatively large and approximately equal efficiency gains with respect to the inverse intertemporal elasticity of substitution γ and the habit in consumption θ_c due to y, c, and i.

Table C4: Information content of asset prices: innovations

	innovation	IG($\bar{\boldsymbol{y}}$)	IG($v^f, r\|\bar{\boldsymbol{y}}$)	IG($v^f\|\bar{\boldsymbol{y}}$)	IG($r\|\bar{\boldsymbol{y}}$)	IG(v^f)	IG(r)
ε_x^u	non-stat. neutral prod.	92.6	5.4	0.2	5.4	3.3	0.3
ε_x^{LR}	non-stat. neutral prod. LR news	0.3	0.1	0.0	0.1	0.1	0.0
ε_a^u	non-stat. investment-specific prod.	99.8	0.1	0.1	0.1	1.7	0.8
ε_a^{LR}	non-stat. investment-specific prod. LR news	78.8	15.0	6.4	10.0	60.5	1.8
ε_z^u	stat. neutral prod.	95.5	3.4	0.1	3.4	1.8	52.7
ε_z^{LR}	stat. neutral prod. LR news	48.0	40.1	1.8	39.7	12.4	31.8
ε_ω^u	stat. investment-specific prod.	93.8	3.6	3.6	0.1	7.5	3.7
ε_ω^{LR}	stat. investment-specific prod. LR news	0.0	0.0	0.0	0.0	0.0	0.0

Note: see the note to Table 1 in the main text. $\bar{\boldsymbol{y}}$ includes the growth rates of output, consumption, investment, *and TFP*, hours worked, and the relative price of investment.

The role of TFP. The results in this section suggest a much greater and more distinct role of asset prices with respect to news shocks in the model of Avdjiev (2016) compared to the SGU model. In particular, v^f and r are found to be considerably more informative than any other observed variable with respect to two of the news shocks – the long-run news components of the non-stationary investment-specific productivity shock (ε_a^{LR}) and the stationary neutral productivity shock (ε_z^{LR}). One possible explanation of this finding is that TFP growth is assumed to be observed in the analysis of the SGU model but not for the model in this section. Since that variable was found to be quite informative with respect to several anticipated innovations in the SGU model, it is possible that the contribution of v^f and r in the model of Avdjiev (2016) is exaggerated by its exclusion. To examine this possibility, Table C4 reevaluates the information content of asset prices assuming that $\bar{\boldsymbol{y}}$ contains tfp in addition to the other five macro variables. The only major change compared to Table C1 is with respect to the long-run news component in the non-stationary investment-specific productivity shock (ε_a^{LR}) and the unanticipated innovation to the stationary investment-specific productivity shock (ε_ω^u). In both cases the conditional contribution of information by asset prices is much smaller when $\bar{\boldsymbol{y}}$ includes tfp. Furthermore, the reduction is almost entirely due to the much smaller contribution of v^f. In the case of ε_a^{LR} the conditional information gain of v^f decreases from 37% to 15%, while at the same time the information gain of r increases from 3.6% to 10%. This implies that there is a negative conditional complementarity between v^f and tfp, and a positive conditional complementarity between r and tfp with respect to ε_a^{LR}. The same type of complementarity between asset prices and tfp is found with respect to the stationary neutral productivity shock (ε_z^{LR}). However, since the relative contribution of v^f is much smaller, the overall information gain of asset prices with respect to that shock remains large.

Figure C3 presents the conditional information gains of all eight variables. r is slightly more informative than tfp, h, v^f and a with respect to ε_a^{LR}; it is, however, by far the most informative variable with respect to ε_a^{LR}. Comparing the results against those presented in Figure C1 shows that the inclusion of tfp has also a significant impact on the contribution of information by h and a. For instance, the conditional gain of h with respect to the non-stationary neutral productivity shock (ε_x^u) declines from 80% when tfp is excluded from $\bar{\boldsymbol{y}}$ to 1% when it is included. This means that, conditional on the other observables, h and tfp are close substitutes in terms of information they contribute about ε_x^u. Similarly, there is a strong negative complementarity between tfp and a with respect to the non-stationary investment-specific productivity shock (ε_a^u).

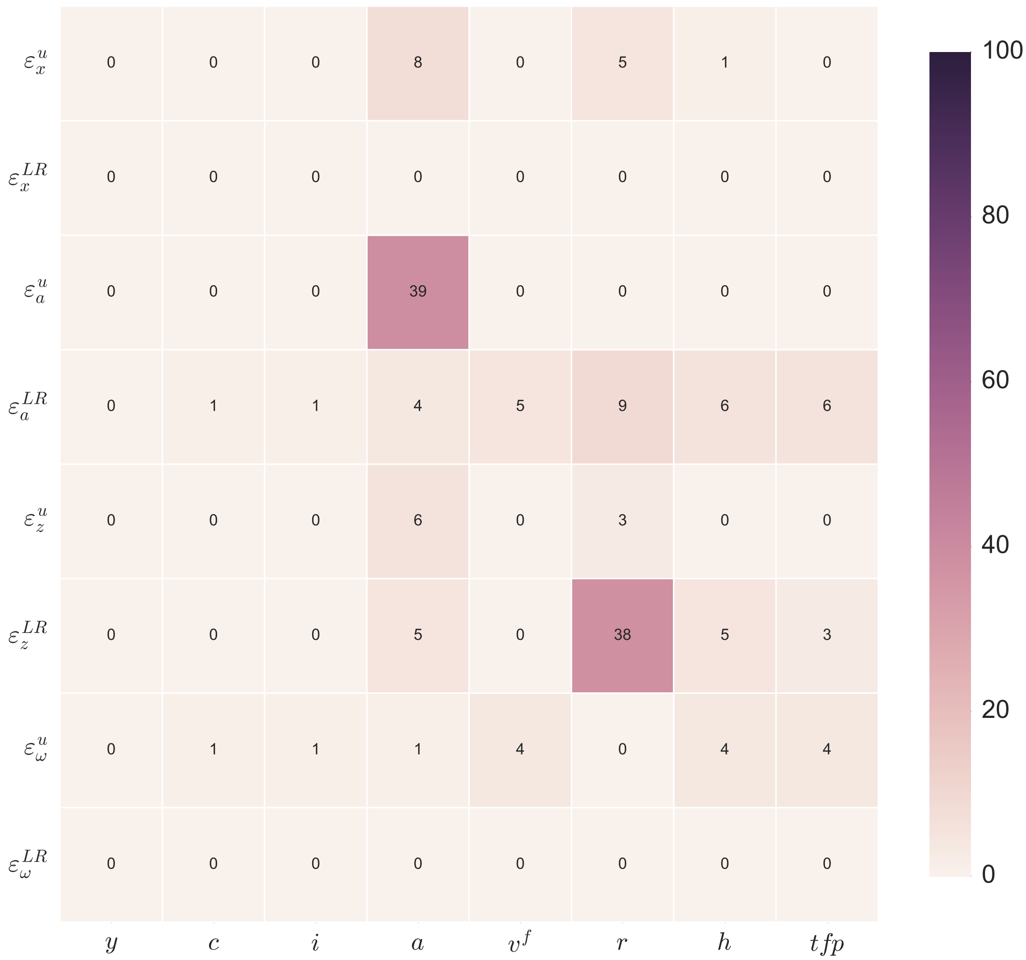

Figure C3: Conditional information gains in the model of Avdjiev (2016) when tfp is observed.

The consequences, in terms of efficiency gains, of adding tfp as an observable are very similar: the contribution of v^f is much smaller than before with respect to most parameters including $\sigma_{a,LR}$, for which it is the most informative variable when tfp is unobserved. The relative importance of r, on the other hand, generally increases, and it becomes the variable with the largest contribution with respect to $\sigma_{z,LR}$. A complete set of results can be seen in Figure C8.

To summarize, when tfp is among the observed variables, of the two asset prices only r contributes significantly more information about one of the news shocks than any other observable. As in Section 4, that shock is the stationary neutral productivity news shock. Due to the relatively smaller number of shocks in the Avdjiev (2016) model, however, the information gained from observing r is substantially larger than in the SGU model.

Table C5: Parameter values, Avdjiev (2016) model

	parameter	value
γ	inverse intertemporal elasticity of substitution	0.90
χ	Frisch elasticity of labor supply	2.90
θ_l	habit in leisure,	0.12
θ_c	habit in consumption,	0.22
δ_2	capacity utilization cost	3.91
η	investment adjustment cost	0.29
ρ_x^l	AR non-stationary neutral productivity	0.01
ρ_a^l	AR non-stationary investment-specific productivity	0.32
ρ_z^l	AR stationary neutral productivity	0.57
ρ_ω^l	AR stationary investment-specific productivity	0.91
$\sigma_{x,u}$	std. non-stationary neutral productivity	1.05
$\sigma_{x,LR}$	std. non-stationary neutral productivity LR news	0.01
$\sigma_{a,u}$	std. non-stationary investment-specific productivity	0.95
$\sigma_{a,LR}$	std. non-stationary investment-specific productivity LR news	0.13
$\sigma_{z,u}$	std. stationary neutral productivity	0.93
$\sigma_{z,LR}$	std. stationary neutral productivity LR news	0.92
$\sigma_{\omega,u}$	std. stationary investment-specific productivity	9.97
$\sigma_{\omega,LR}$	std. stationary investment-specific productivity LR news	0.07

Note: The values are the posterior median estimates reported in Table D.2 of Avdjiev (2016).

Table C6: Cramér-Rao lower bounds, Avdjiev (2016) model.

	parameter	$\bar{\boldsymbol{y}}$	$\boldsymbol{y}$
γ	inverse intertemporal elasticity of substitution	0.00024	0.00020
χ	Frisch elasticity of labor supply	443.84281	6.00170
θ_l	habit in leisure,	0.00995	0.00601
θ_c	habit in consumption,	0.00006	0.00004
δ_2	capacity utilization cost	1057.97045	128.57305
η	investment adjustment cost	0.00609	0.00071
ρ_x^l	AR non-stationary neutral productivity	0.00439	0.00066
ρ_a^l	AR non-stationary investment-specific productivity	0.00461	0.00371
ρ_z^l	AR stationary neutral productivity	0.01160	0.00117
ρ_ω^l	AR stationary investment-specific productivity	0.00201	0.00097
$\sigma_{x,u}$	std. non-stationary neutral productivity	0.00548	0.00370
$\sigma_{x,LR}$	std. non-stationary neutral productivity LR news	0.00003	0.00002
$\sigma_{a,u}$	std. non-stationary investment-specific productivity	0.00206	0.00196
$\sigma_{a,LR}$	std. non-stationary investment-specific productivity LR news	0.00086	0.00027
$\sigma_{z,u}$	std. stationary neutral productivity	0.00391	0.00214
$\sigma_{z,LR}$	std. stationary neutral productivity LR news	0.02206	0.00790
$\sigma_{\omega,u}$	std. stationary investment-specific productivity	8.20461	0.79912
$\sigma_{\omega,LR}$	std. stationary investment-specific productivity LR news	965.05866	132.27654

Note: $\boldsymbol{y}$ includes all observables, $\bar{\boldsymbol{y}} = \boldsymbol{y} \setminus (v^f, r)$ The CRLBs are computed for the parameter values in Table C5 using $T = 236$.

	y	c	i	a	h
ε_x^u	0	0	-0.01	-0.03	-0.01
ε_x^{LR}	0	0	0	0.01	-0.01
ε_a^u	0	0	0	-0.03	0
ε_a^{LR}	0	0	0	-0.03	0
ε_z^u	0	0.08	0.02	-0.01	0.01
ε_z^{LR}	0	0	0	-0.15	0.13
ε_ω^u	0	0	0	0.09	-0.01
ε_ω^{LR}	0	0	0	0.16	0

Figure C4: Conditional pairwise complementarity between v^f and macro variables Avdjiev (2016)

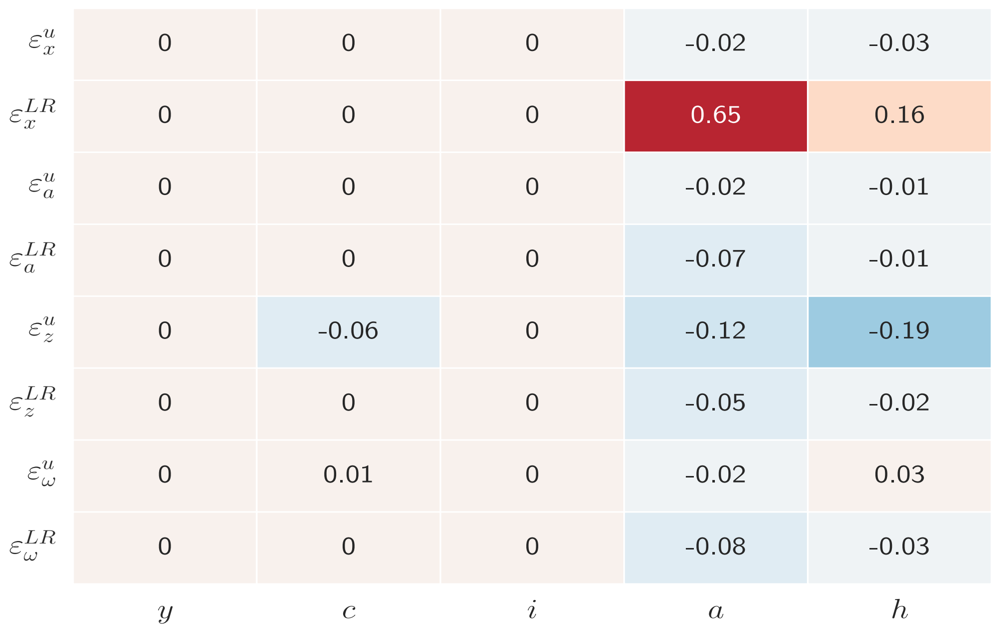

Figure C5: Conditional pairwise complementarity between r and macro variables
Avdjiev (2016)

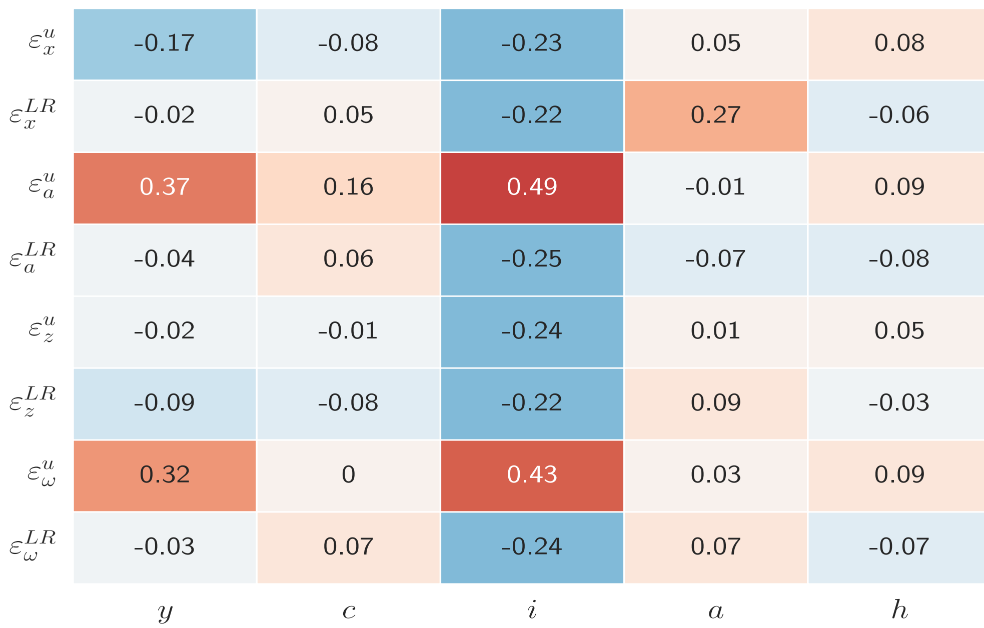

Figure C6: Unconditional pairwise complementarity between v^f and macro variables
Avdjiev (2016)

	y	c	i	a	h
ε_x^u	0.37	1	0.21	0.01	0.19
ε_x^{LR}	0.28	-0.19	-0.07	2.3	0.13
ε_a^u	0.22	0.28	0.22	-0.01	0.24
ε_a^{LR}	0.94	0.18	0	-0.14	0.14
ε_z^u	-0.32	-0.35	0.1	0.01	-0.1
ε_z^{LR}	0.19	0.33	-0.11	0.02	-0.15
ε_ω^u	0.69	-0.37	-0.06	0.07	-0.02
ε_ω^{LR}	0.93	0.4	0.02	0.4	0.16

Figure C7: Unconditional pairwise complementarity between r and macro variables Avdjiev (2016)

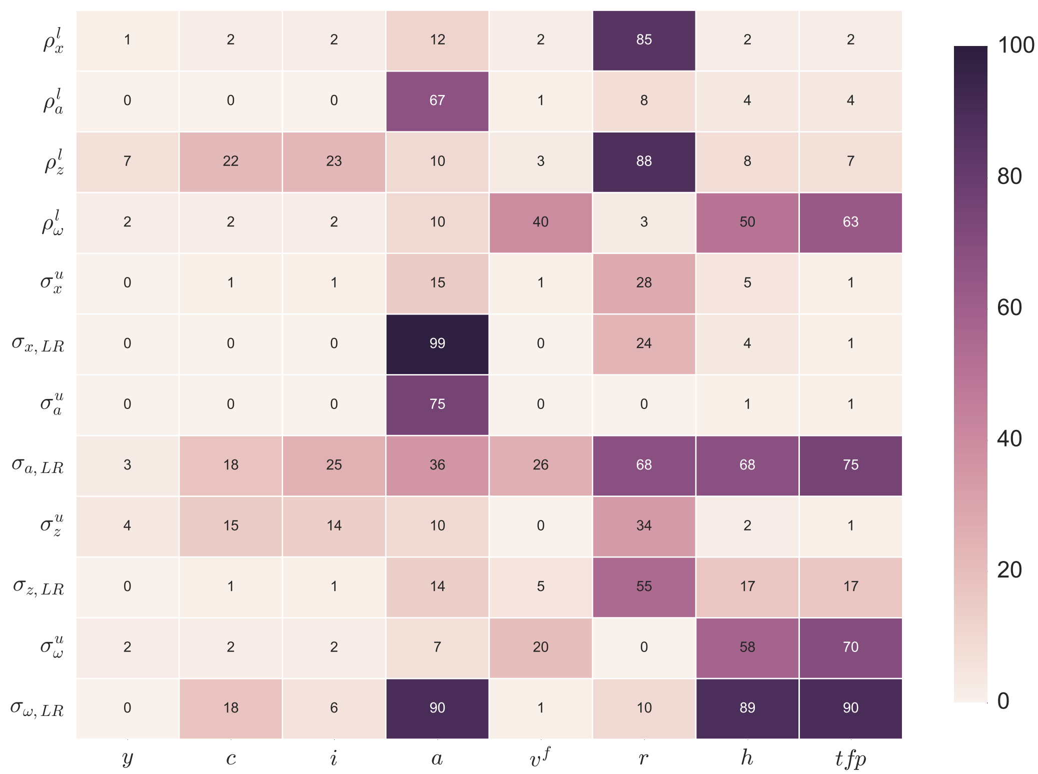

Figure C8: Efficiency gains in the model of Avdjiev (2016) when tfp is observed.

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