NOT-FOR-PUBLICATION APPENDIX: On the sources of information in the moment structure of dynamic macroeconomic models

A Schmitt-Grohé and Uribe (2012) model

A.1 The Model

The model economy is populated by a continuum of identical agents each maximizing the following lifetime utility function

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \frac{\left[C_t - bC_{t-1} - \psi H_t^{\theta} S_t\right]^{1-\sigma} - 1}{1-\sigma}, \tag{A.1}$$

where ζ_t is a preference shock, C_t is consumption, H_t is hours worked, and S_t is a geometric average of past habit-adjusted consumption: $S_t = (C_t - bC_{t-1})^{\gamma} S_{t-1}^{1-\gamma}$. The household budget constraint is given by

$$C_t + A_t I_t + T_t = W_t H_t + r_t u_t K_t + P_t, \tag{A.2}$$

where A_t is a non-stationary investment specific productivity growing at rate μ_t^a. The variable T_t denotes lump-sum taxes, W_t is the wage rate, r_t is rental rate of capital, u_t is capacity utilization, K_t is capital stock, and P_t denotes profit. The law of motion for capital stock is

$$K_{t+1} = (1 - \delta(u_t))K_t + z_t^I I_t \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \mu^I\right)\right], \tag{A.3}$$

where I_t is investment, δ is the rate of depreciation – an increasing function of the rate of capacity utilization u_t, κ is a parameter that determines the convexity of the investment adjustment cost function, μ^I is is the steady state growth rate of investment, and z_t^I is a stationary investment specific productivity shock.

Final good Y_t is produced with the following production function:

$$Y_t = z_t (u_t K_t)^{\alpha_k} (X_t H_t)^{\alpha_h} (X_t L)^{1-\alpha_k-\alpha_h}, \tag{A.4}$$

where z_t is a stationary neutral productivity shock, L is a fixed factor of production,[1] and X_t is a non-stationary neutral productivity growing at rate μ_t^x.

The labor input H_t, which is used by final-good-producing firms, is obtained by combining differentiated labor services H_{jt} supplied by monopolistically competitive labor unions,

$$H_t = \left[\int_0^1 H_{jt}^{\frac{1}{1+\mu_t}} dj\right]^{1+\mu_t}, \tag{A.5}$$

where μ_t is a wage markup shock with steady state value $\mu > 1$.

Each period the government spends an amount G_t, financed with lump-sum taxes.

[1] The fixed factor of production generates decreasing returns to scale in the two variable factors of production K_t and H_t. As shown by Jaimovich and Rebelo (2009) this allows for a positive response of the value of the firm to future expected increases in productivity.

G_t is determined exogenously and is assumed to grow at rate X_t^G, defined as a smoothed version of the trend in Y_t, given by $X_t^Y = X_t A_t^{\alpha_k/(\alpha_k-1)}$.

Each of the seven shocks is driven by three independent innovations, two anticipated and one unanticipated. More precisely, the process governing shock x_t for $x = \mu^a, \mu^x, z^I, z, \mu, g, \zeta$ is given by

$$\ln(x_t/x) = \rho_x \ln(x_{t-1}/x) + \sigma_x^0 \varepsilon_{x,t}^0 + \sigma_x^4 \varepsilon_{x,t-4}^4 + \sigma_x^8 \varepsilon_{x,t-8}^8, \tag{A.6}$$

where $\varepsilon_{x,t}^j$ for $j = 0, 4, 8$ are independent standard normal random variables.

SGU report results based on estimation of the model using quarterly data on seven macroeconomic series: the growth rate of per capita real GDP ($y_t = \triangle \ln Y_t$) contaminated with a measurement error, the growth rates of real consumption ($c_t = 100 \triangle \ln C_t$), real investment ($i_t = 100 \triangle \ln A_t I_t$), real government expenditure ($g_t = 100 \triangle \ln G_t$), and hours ($h_t = 100 \triangle \ln H_t$), and the growth rates of the relative price of investment ($a_t = 100 \triangle \ln A_t$) and of total factor productivity ($tfp_t = 100 \triangle \ln TFP_t$).[2]

In addition to these variables, the model makes predictions about the behavior of two asset price variables: the value of the firm and the risk-free real interest rate. The value of the firm V^F can be computed as

$$V_t^F = Y_t - W_t H_t - A_t I_t + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^F, \tag{A.7}$$

where Λ_t is the Lagrange multiplier associated with the household's budget constraint. The risk-free real interest rate is given by

$$R_t = \frac{1}{\beta} \frac{\Lambda_t}{E_t \Lambda_{t+1}}. \tag{A.8}$$

In estimation, the value of the firm can be matched to stock price data. In particular, $v_t^f = \triangle \ln V_t^F$ can be represented with the growth rate of the real per capita value of the stock market. Similarly, data on $r_t = \log R_t$ can be obtained by deflating the nominal rate on the three-month Treasury bill by the inflation rate implied by the GDP deflator.

A.2 Sources of information

This section presents results on the sources of information with respect to the parameters which were excluded from the analysis in the main text. There are two groups of parameters: nine standard deviations of anticipated and unanticipated innovations to three shocks – wage markup (μ), government spending (g) and preference (ζ), and the autoregressive coefficients of all seven of the model shocks. We present the results as in the main section: by lag structure, observables, groups of covariances, and individual moments.

[2]The growth rate of total factor productivity in the model is given by $tfp_t = 100\,(\triangle \ln z_t + (1-\alpha_k) \ln \mu_t^x)$.

Table B1: Parameter values, SGU (2012) model

	parameter	MLE	posterior median
θ	Frisch elasticity of labor supply	5.39	4.74
γ	wealth elasticity of labor supply	0.00	0.00
κ	investment adjustment cost	25.07	9.11
δ_2/δ_1	capacity utilization cost	0.44	0.34
b	habit in consumption	0.94	0.91
ρ_{xg}	government spending	0.74	0.72
ρ_z	AR stationary neutral productivity	0.96	0.92
ρ_{μ^a}	AR non-stationary investment-specific productivity	0.48	0.48
ρ_g	AR government spending	0.96	0.96
ρ_{μ^x}	AR non-stationary neutral productivity	0.27	0.38
ρ_μ	AR wage markup	0.98	0.98
ρ_ζ	AR preference	0.10	0.17
ρ_{z^I}	AR stationary investment-specific productivity	0.21	0.47
σ_z^0	std. stationary neutral productivity 0	0.62	0.65
σ_z^4	std. stationary neutral productivity 4	0.11	0.11
σ_z^8	std. stationary neutral productivity 8	0.11	0.09
$\sigma_{\mu^a}^0$	std. non-stationary investment-specific productivity 0	0.16	0.21
$\sigma_{\mu^a}^4$	std. non-stationary investment-specific productivity 4	0.20	0.16
$\sigma_{\mu^a}^8$	std. non-stationary investment-specific productivity 8	0.19	0.16
σ_g^0	std. government spending 0	0.53	0.62
σ_g^4	std. government spending 4	0.69	0.57
σ_g^8	std. government spending 8	0.43	0.37
$\sigma_{\mu^x}^0$	std. non-stationary neutral productivity 0	0.45	0.38
$\sigma_{\mu^x}^4$	std. non-stationary neutral productivity 4	0.12	0.08
$\sigma_{\mu^x}^8$	std. non-stationary neutral productivity 8	0.12	0.10
σ_μ^0	std. wage markup 0	1.51	0.50
σ_μ^4	std. wage markup 4	3.93	4.79
σ_μ^8	std. wage markup 8	3.20	0.51
σ_ζ^0	std. preference 0	2.83	4.03
σ_ζ^4	std. preference 4	2.76	1.89
σ_ζ^8	std. preference 8	5.34	2.21
$\sigma_{z^I}^0$	std. stationary investment-specific productivity 0	34.81	11.72
$\sigma_{z^I}^4$	std. stationary investment-specific productivity 4	11.99	1.93
$\sigma_{z^I}^8$	std. stationary investment-specific productivity 8	14.91	5.50

Note: The values are taken from Table II of Schmitt-Grohé and Uribe (2012)

Lag structure

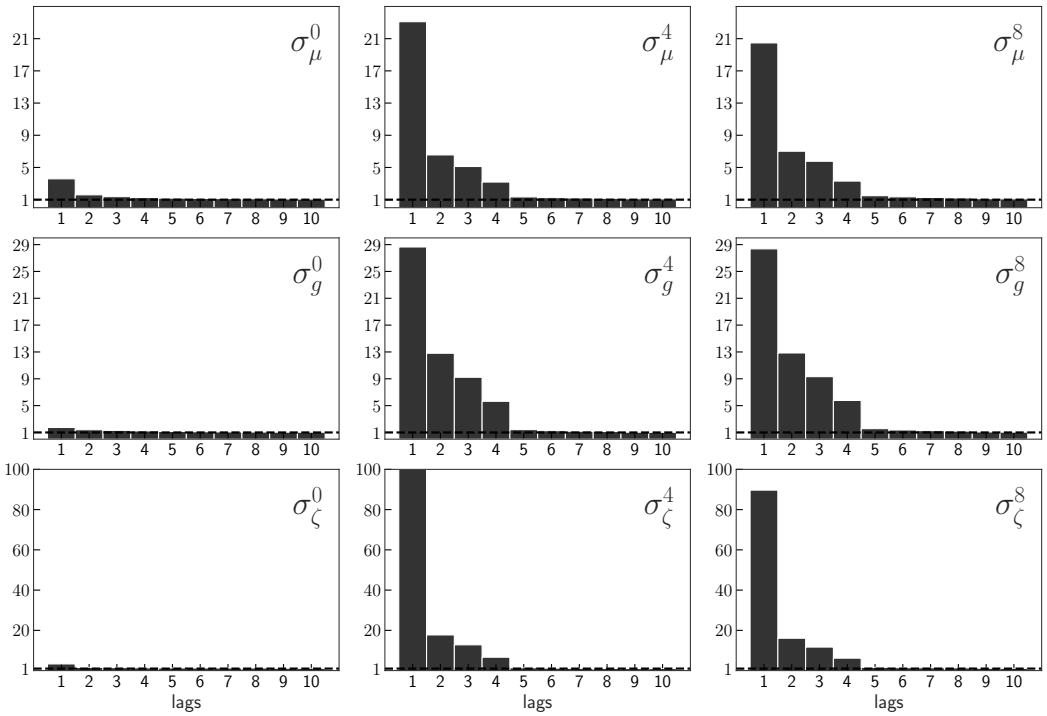

Figure B1: Wage markup (μ), government spending (g) and preference (ζ) shocks parameters. The figure shows the asymptotic efficiency of GMM estimator for different number of lags relative to MLE.

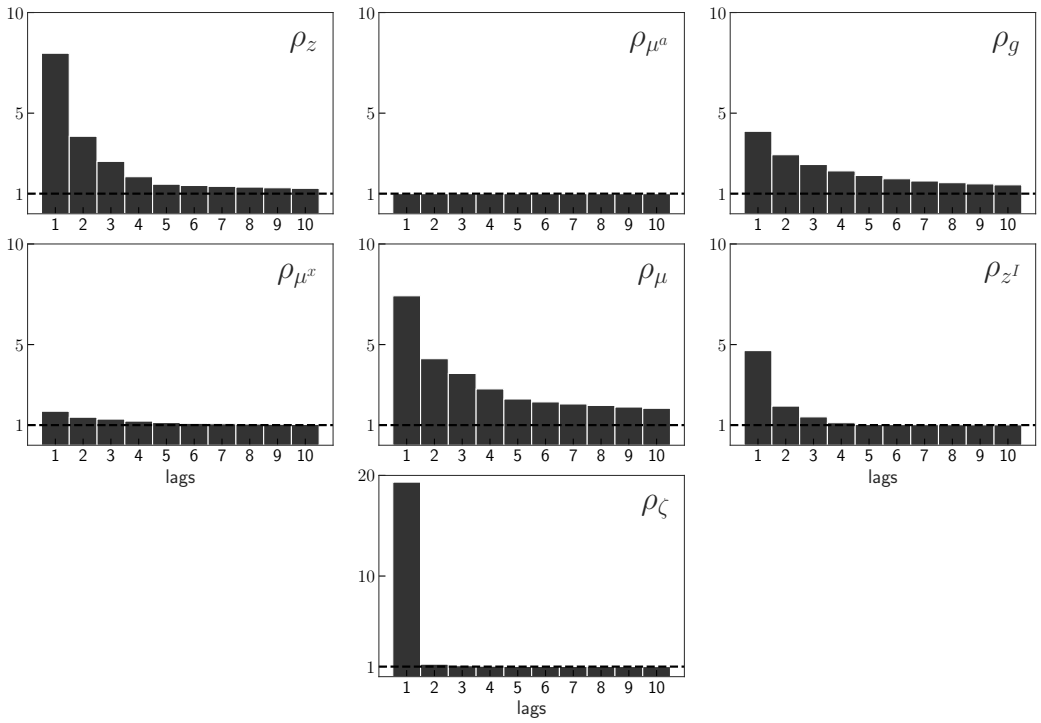

Figure B2: Autoregressive coefficients of the shock processes. The figure shows the asymptotic efficiency of GMM estimator for different number of lags relative to MLE.

Observed variables

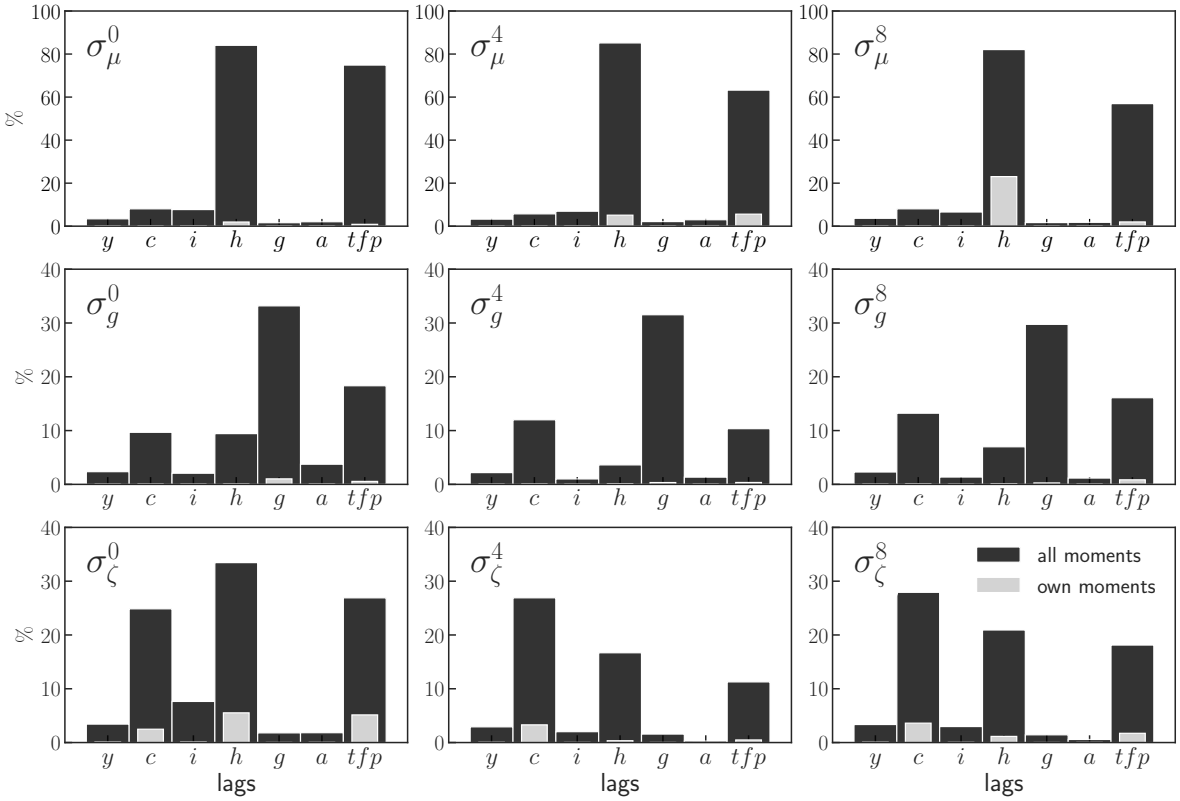

Figure B3: Wage markup (μ), government spending (g) and preference (ζ) shocks parameters. The figure shows the efficiency gains due to moments of each observed variable.

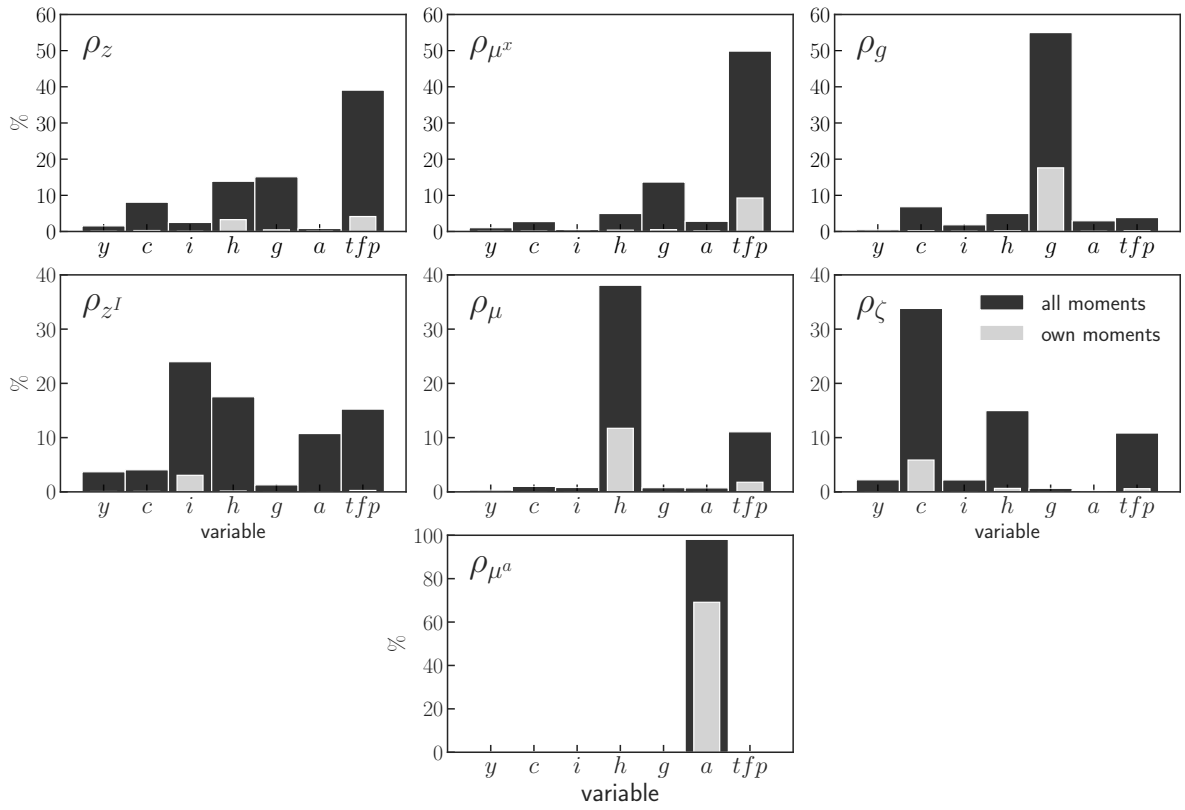

Figure B4: Autoregressive coefficients of the shock processes. The figure shows the efficiency gains due to moments of each observed variable.

Main groups of covariances

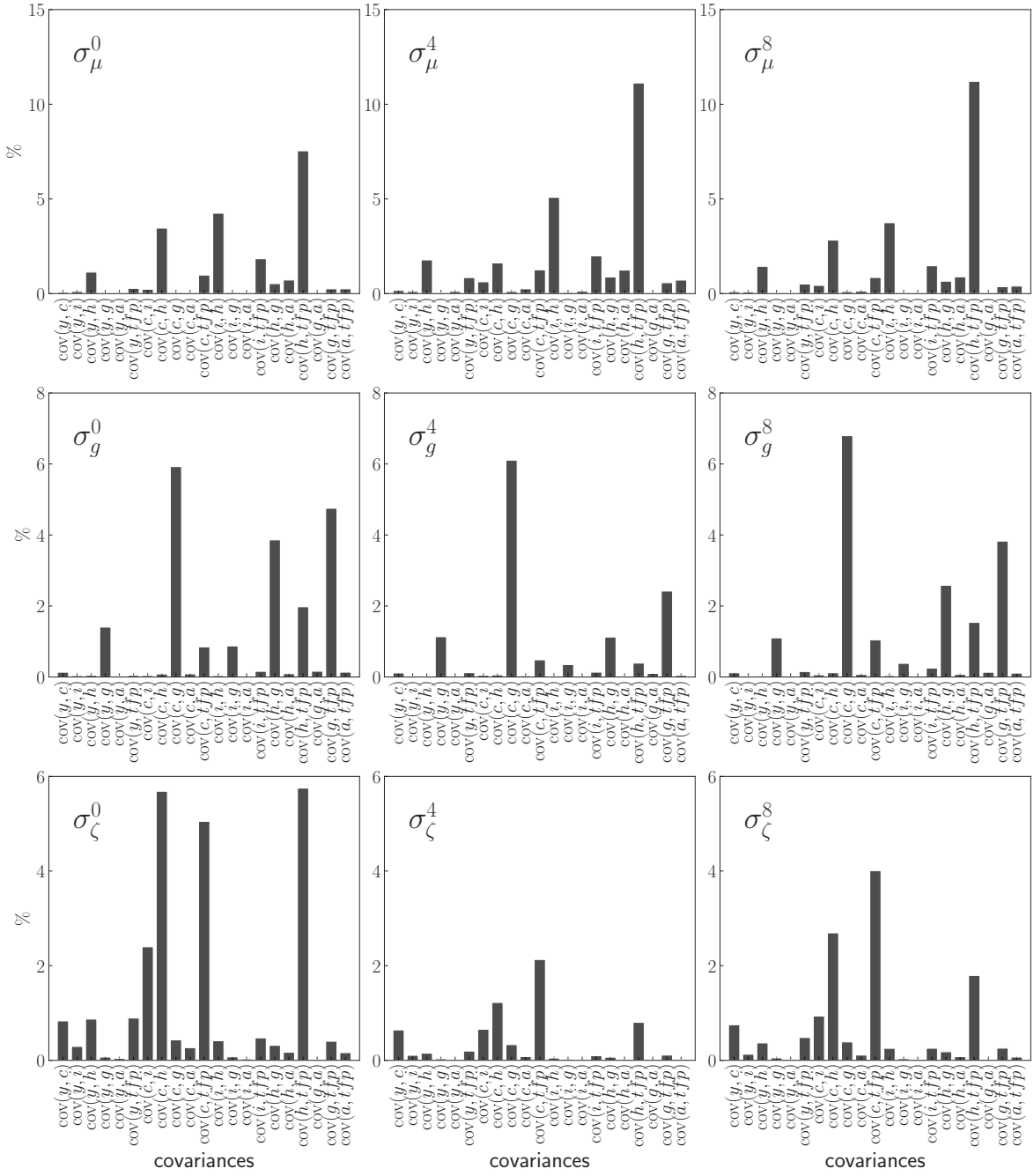

Figure B5: Wage markup (μ), government spending (g) and preference (ζ) shocks parameters.
The figure shows the efficiency gains due the covariances in each group.

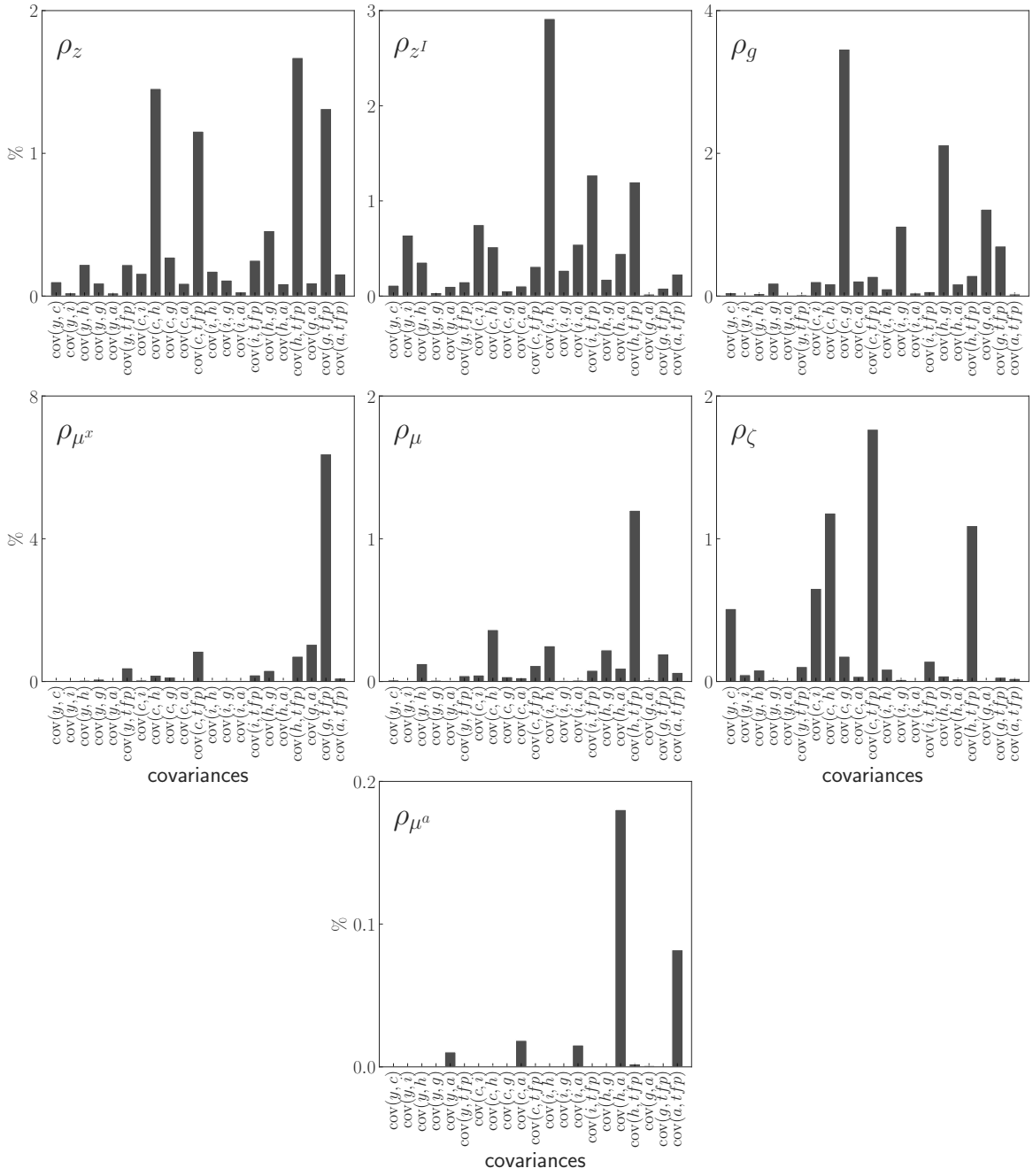

Figure B6: Autoregressive coefficients of the shock processes. The figure shows the efficiency gains due the covariances in each group.

Individual moments

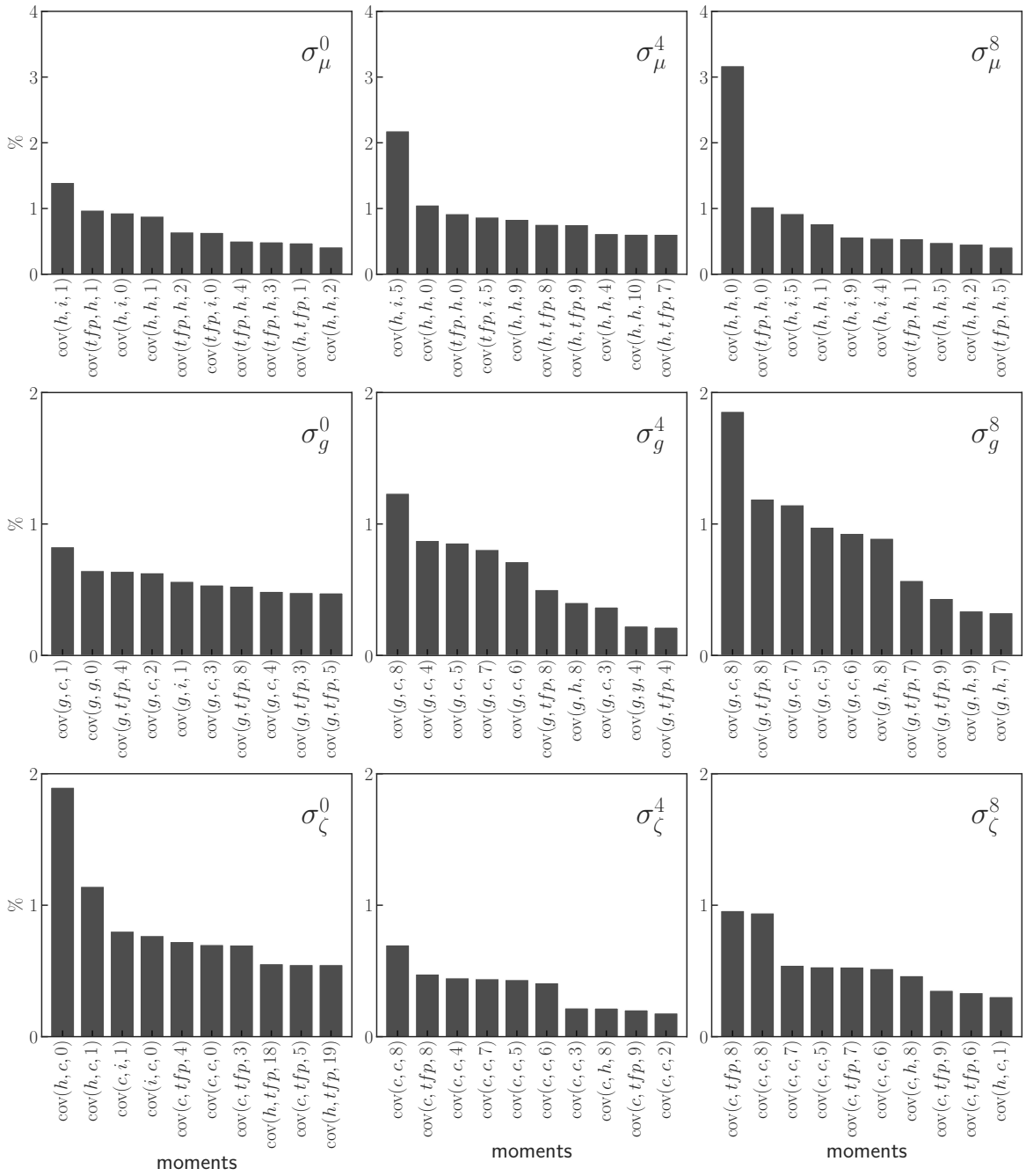

Figure B7: Wage markup (μ), government spending (g) and preference (ζ) shocks parameters. The figure shows the 10 most informative moments with respect to each parameter.

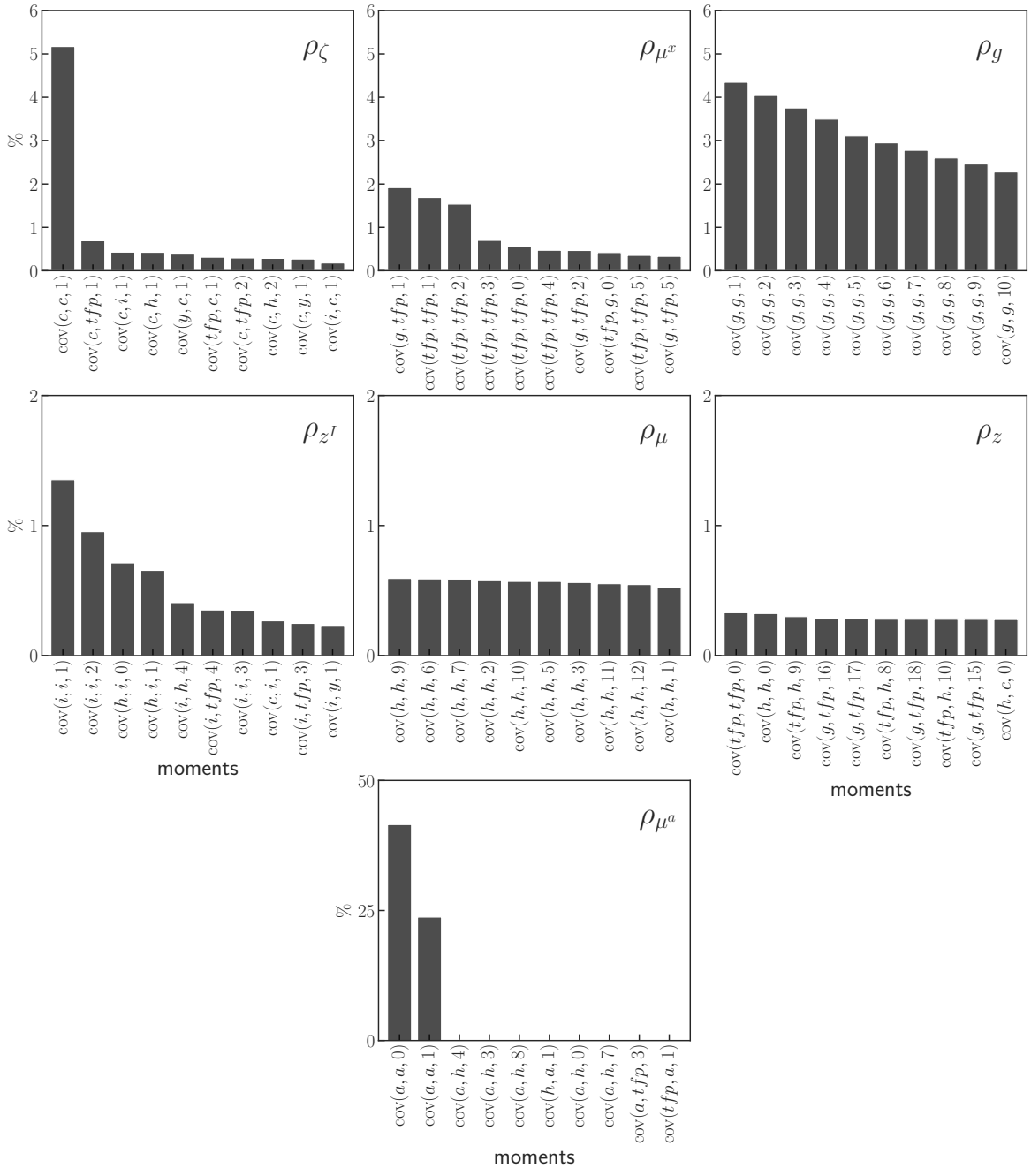

Figure B8: Autoregressive coefficients of the shock processes. The figure shows the 10 most informative moments with respect to each parameter.

B Smets and Wouters (2007) model

In this section we present results from applying the methodology of the paper to the DSGE model estimated in Smets and Wouters (2007), which is a medium-scale closed-economy New Keynesian model featuring price and wage rigidities, habit formation, capital accumulation, investment adjustment cost, variable capital utilization. The model is estimated using US quarterly data ranging from 1966:Q1 to 2004:Q4. The data set includes output growth (y), consumption growth (c), investment growth (i), real wage growth (w), hours worked (h), inflation (π and the nominal interest rate (r). The parameters we analyze are shown in Table B1. To organize the results, these parameters are divided into five groups: price-wage block parameters (price and wage markup, price and wage indexation, price and wage stickiness), preference and technology parameters (capacity utilization cost, investment adjustment cost, elasticity of intertemporal substitution, habit persistence, fixed cost in production, elasticity of labor supply), monetary policy parameters (monetary policy response to inflation, monetary policy response to output gap, monetary policy response to change in output gap, interest rate smoothing, persistence of monetary policy shock, volatility of monetary policy shock), and two groups of shock parameters - (1) productivity, risk premium, government spending shock persistence and volatility parameters, and (2) investment-specific, price and wage markup shock persistence and volatility parameters. The results are organized as in the main text: by lag structure, observables, groups of covariances, and individual moments.

B.1 Lag structure

Table B1: Estimated parameters, SW (2007) model

parameter	
μ_w	MA wage markup
μ_p	MA price markup
ψ	capacity utilization cost
φ	investment adjustment cost
σ_c	elasticity of intertemporal substitution
λ	habit
Φ	fixed cost in production
ι_w	wage indexation
ξ_w	wage stickiness
ι_p	price indexation
ξ_p	price stickiness
σ_l	elasticity of labor supply
r_π	monetary policy response to inflation
$r_{\triangle y}$	monetary policy response to change in output gap
r_y	monetary policy response to output gap
ρ	interest rate smoothing
ρ_a	AR productivity shock
ρ_b	AR risk premium shock
ρ_g	AR government spending shock
ρ_I	AR investment specific shock
ρ_r	AR monetary policy shock
ρ_p	AR price markup shock
ρ_w	AR wage markup shock
γ	trend growth rate
σ_a	standard deviation productivity shock
σ_b	standard deviation risk premium shock
σ_g	standard deviation government spending shock
σ_I	standard deviation investment specific shock
σ_r	standard deviation monetary policy shock
σ_p	standard deviation price markup shock
σ_w	standard deviation wage markup shock

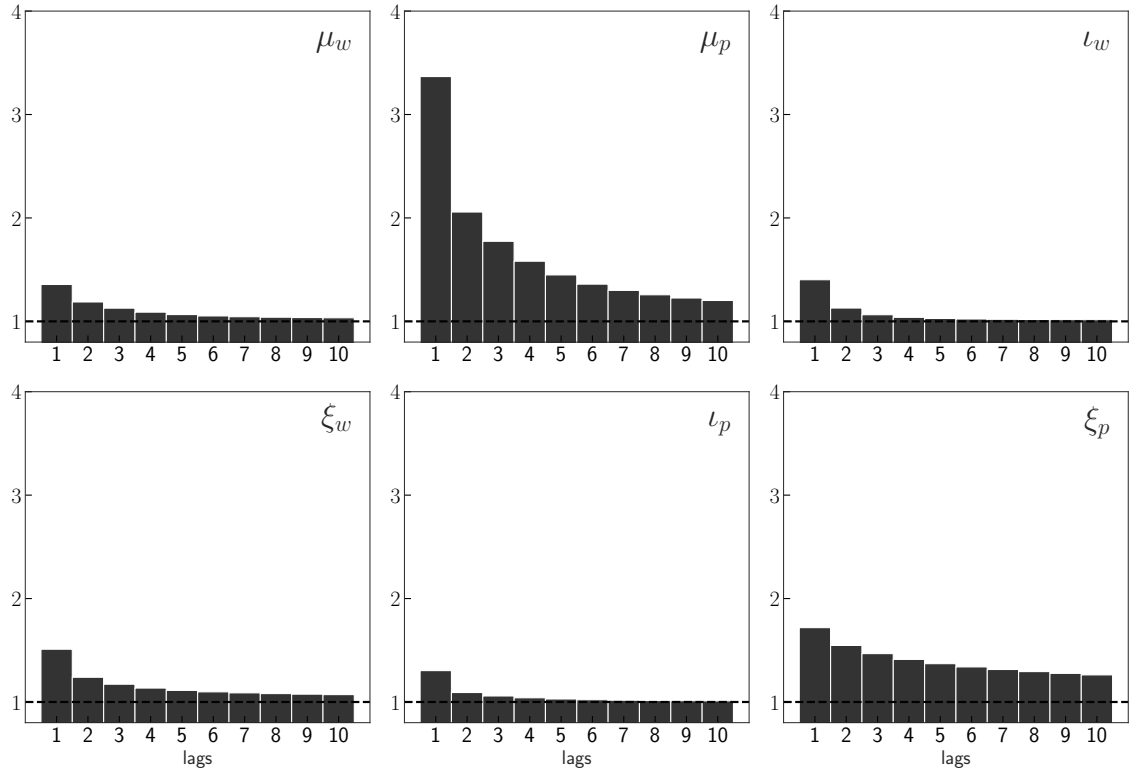

Figure B1: Price-wage block parameters. The figure shows the asymptotic efficiency of GMM estimator for different number of lags relative to MLE.

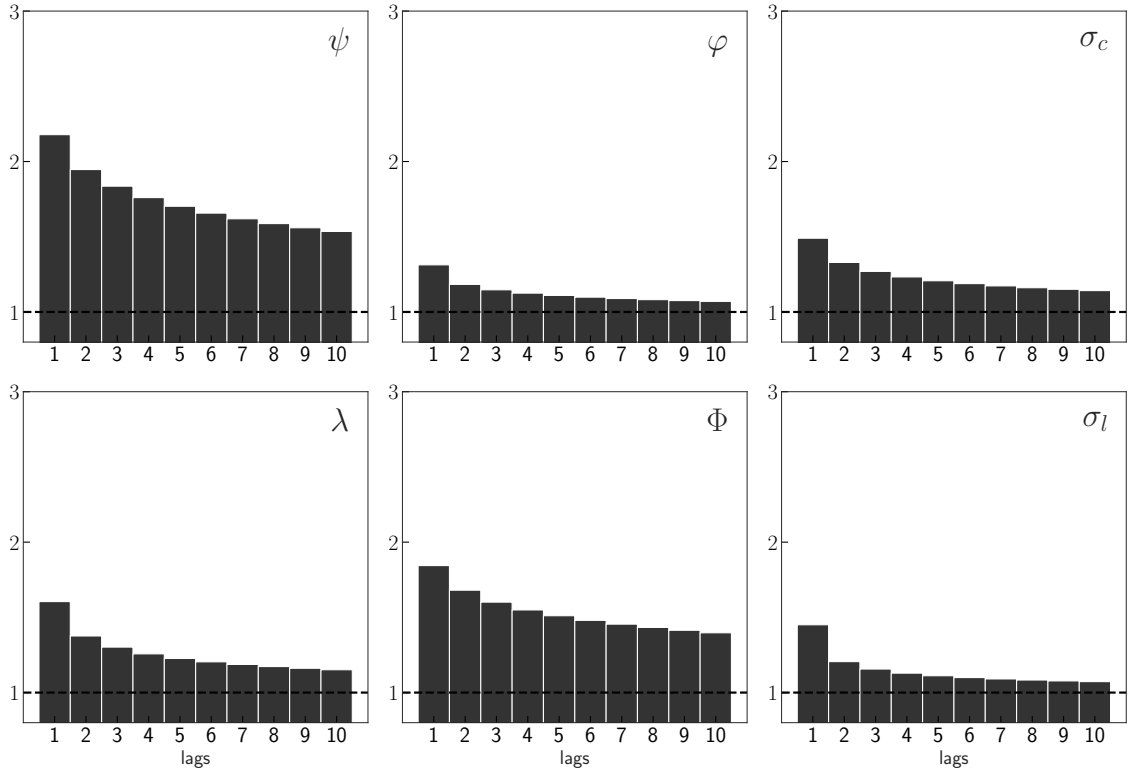

Figure B2: Preference and technology parameters. The figure shows the asymptotic efficiency of GMM estimator for different number of lags relative to MLE.

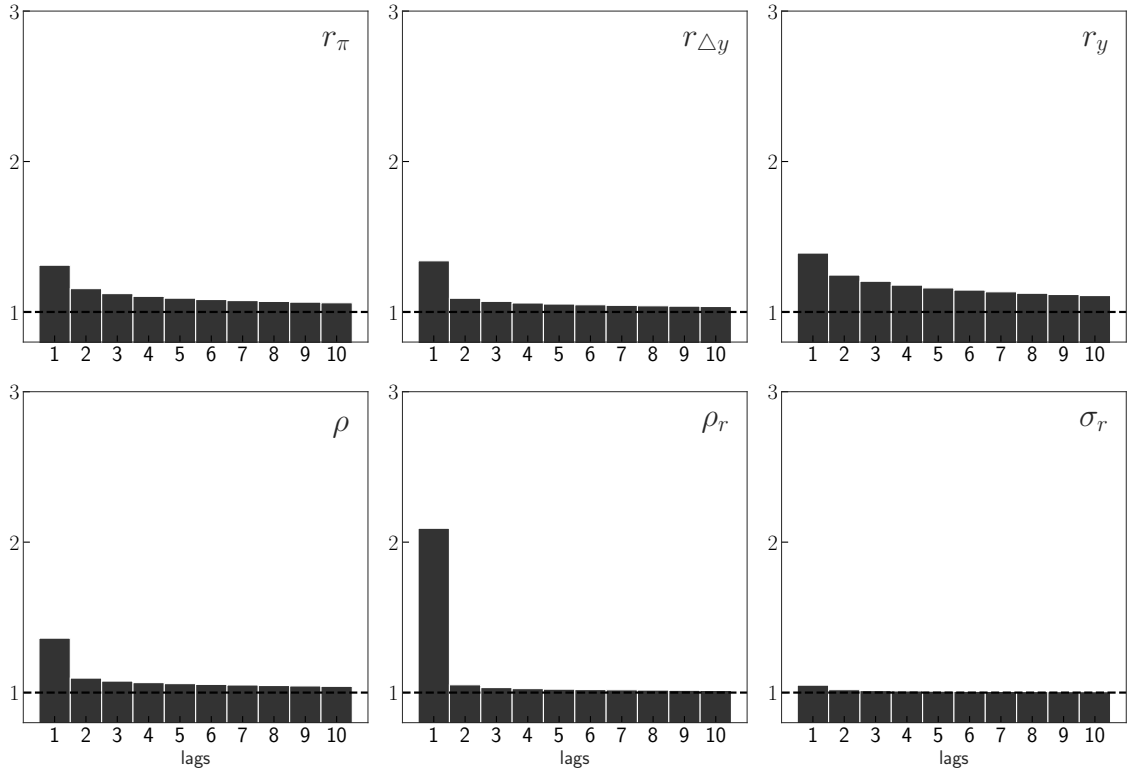

Figure B3: Monetary policy parameters. The figure shows the asymptotic efficiency of GMM estimator for different number of lags relative to MLE.

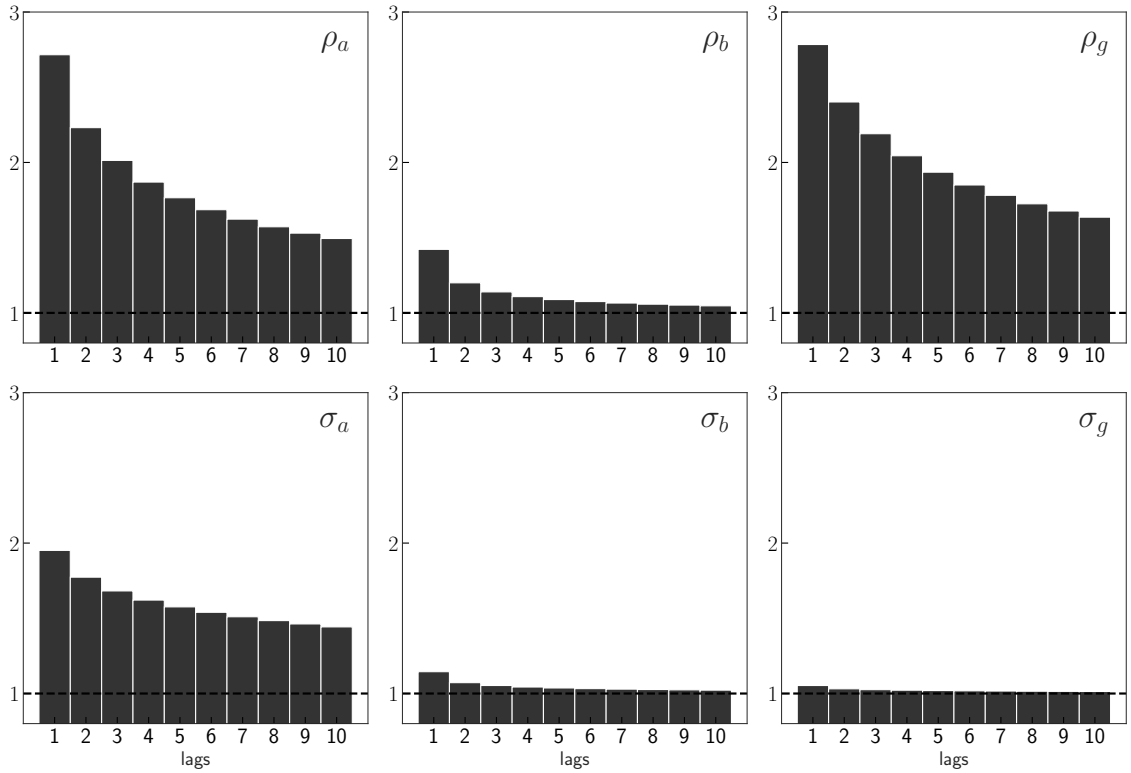

Figure B4: Productivity, risk premium, government spending shock parameters. The figure shows the asymptotic efficiency of GMM estimator for different number of lags relative to MLE.

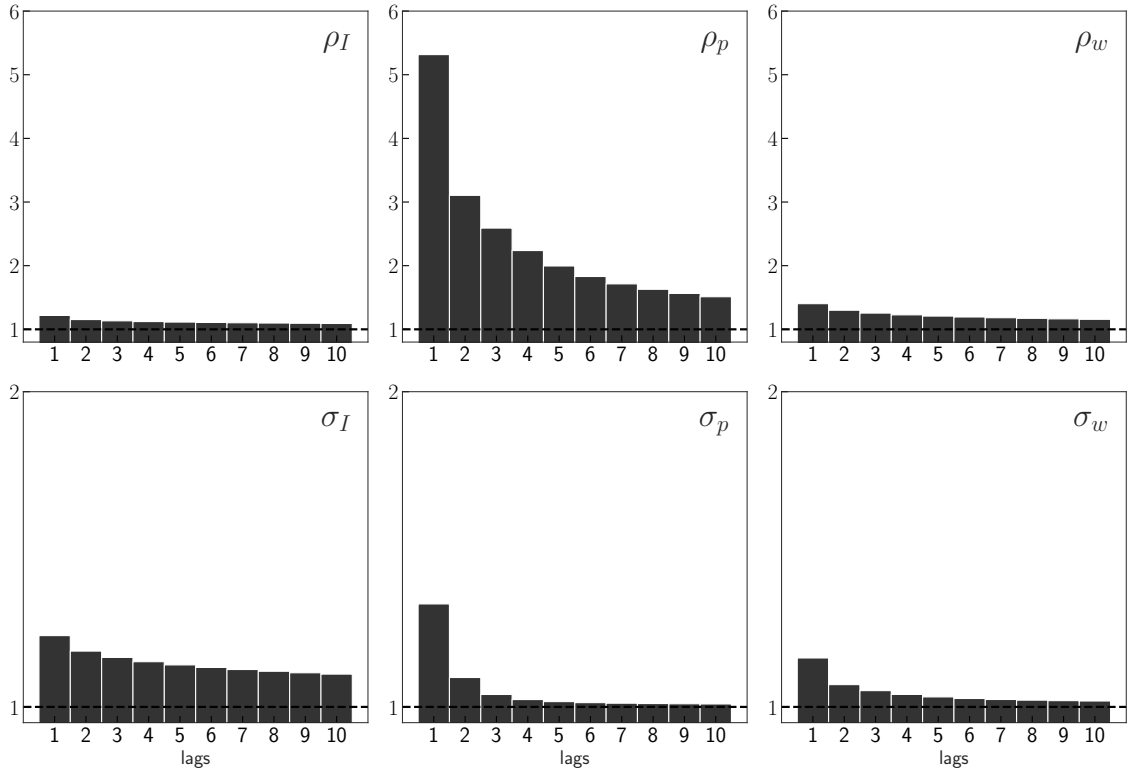

Figure B5: Investment-specific, price and wage markup shock parameters. The figure shows the asymptotic efficiency of GMM estimator for different number of lags relative to MLE.

B.2 Observed variables

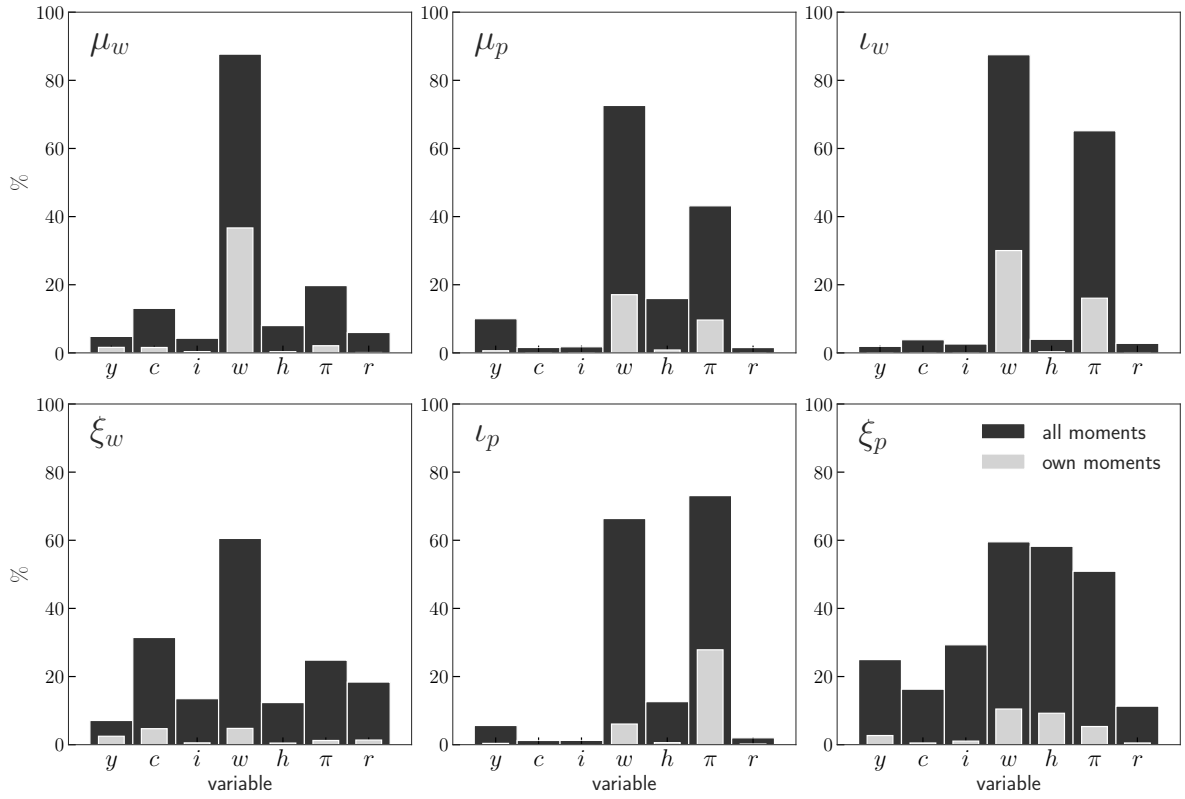

Figure B6: Price-wage block parameters. The figure shows the efficiency gains due to moments of each observed variable.

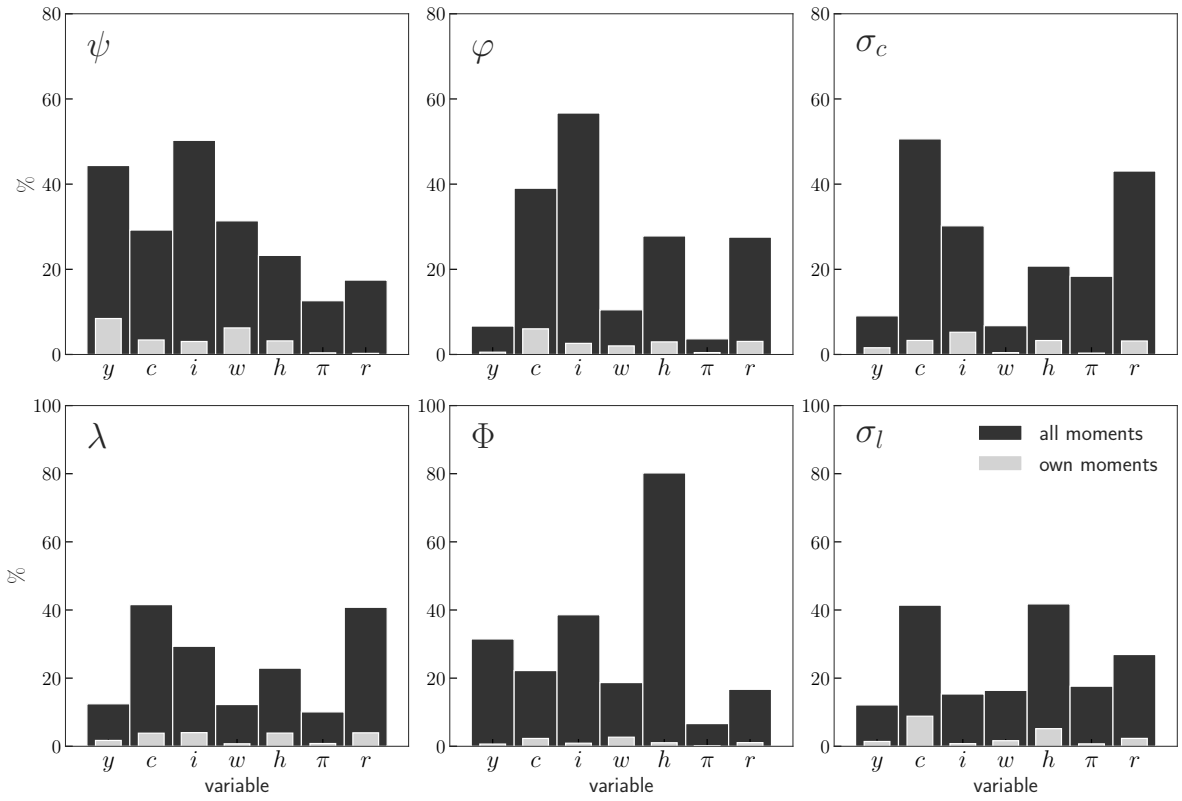

Figure B7: Preference and technology parameters. The figure shows the efficiency gains due to moments of each observed variable.

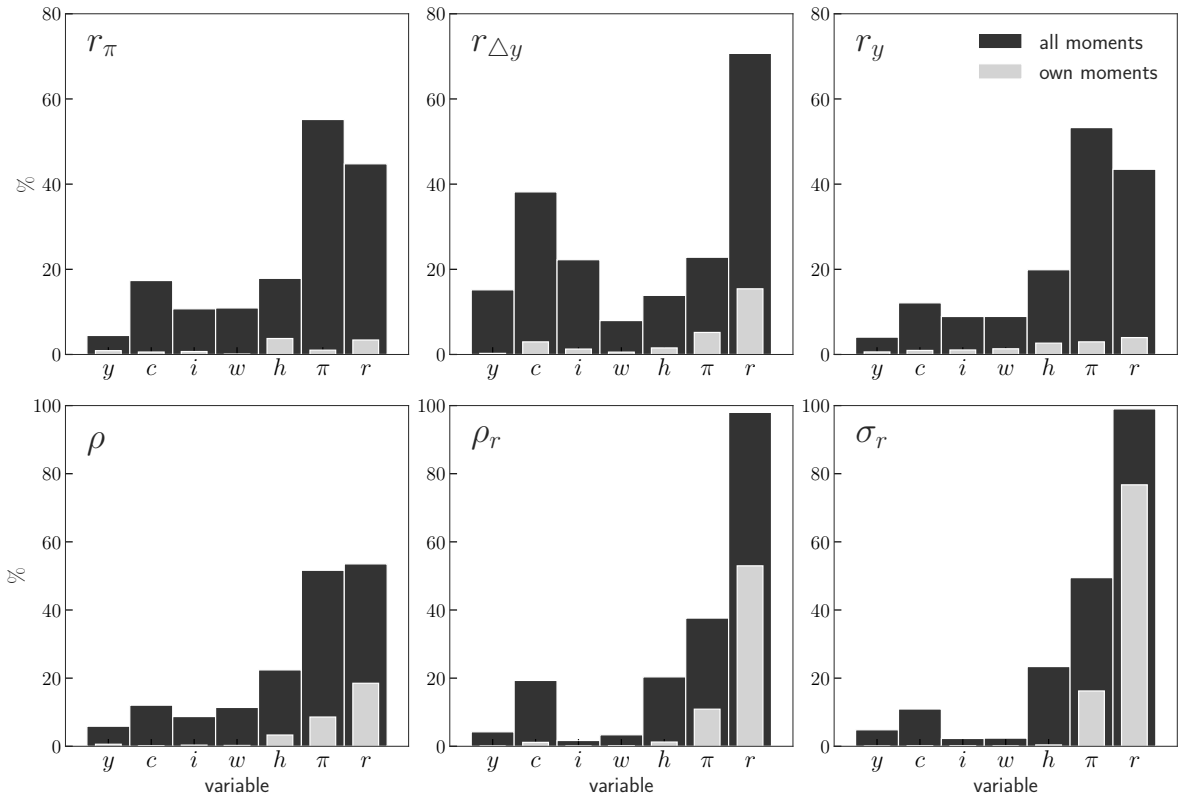

Figure B8: Monetary policy parameters. The figure shows the efficiency gains due to moments of each observed variable.

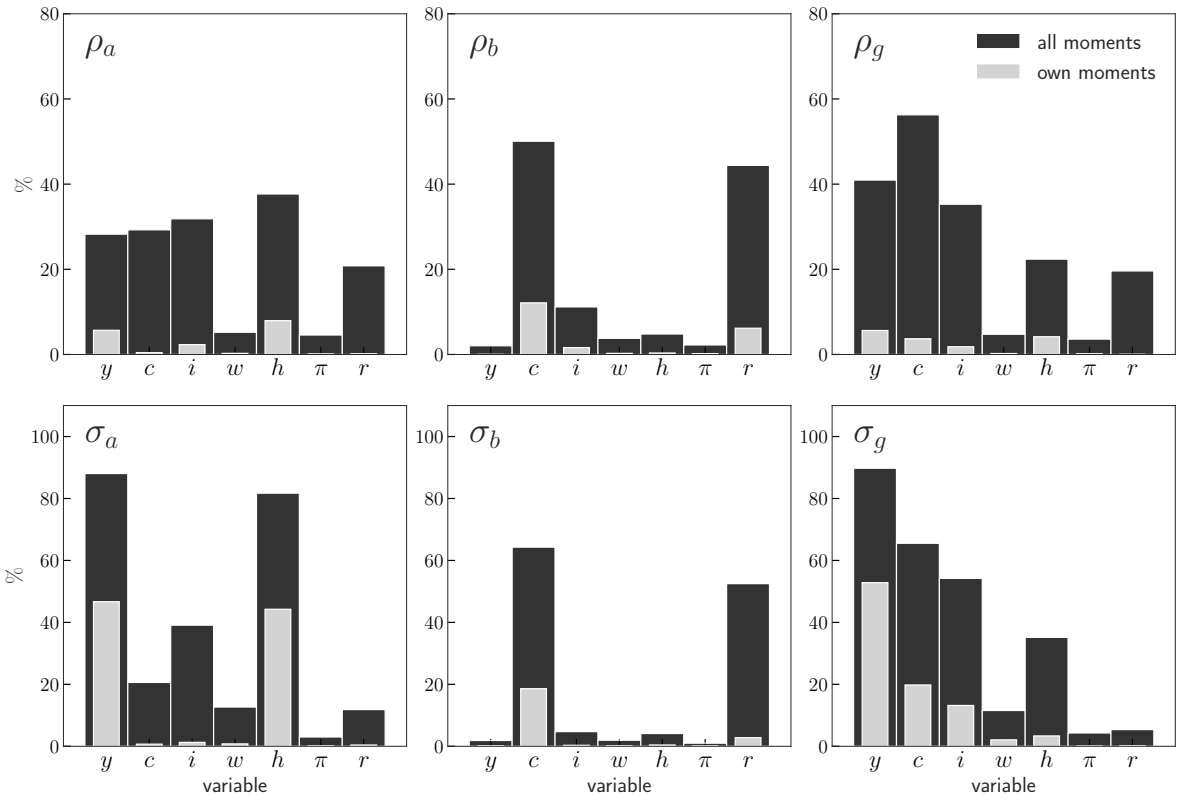

Figure B9: Productivity, risk premium, government spending shock parameters. The figure shows the efficiency gains due to moments of each observed variable.

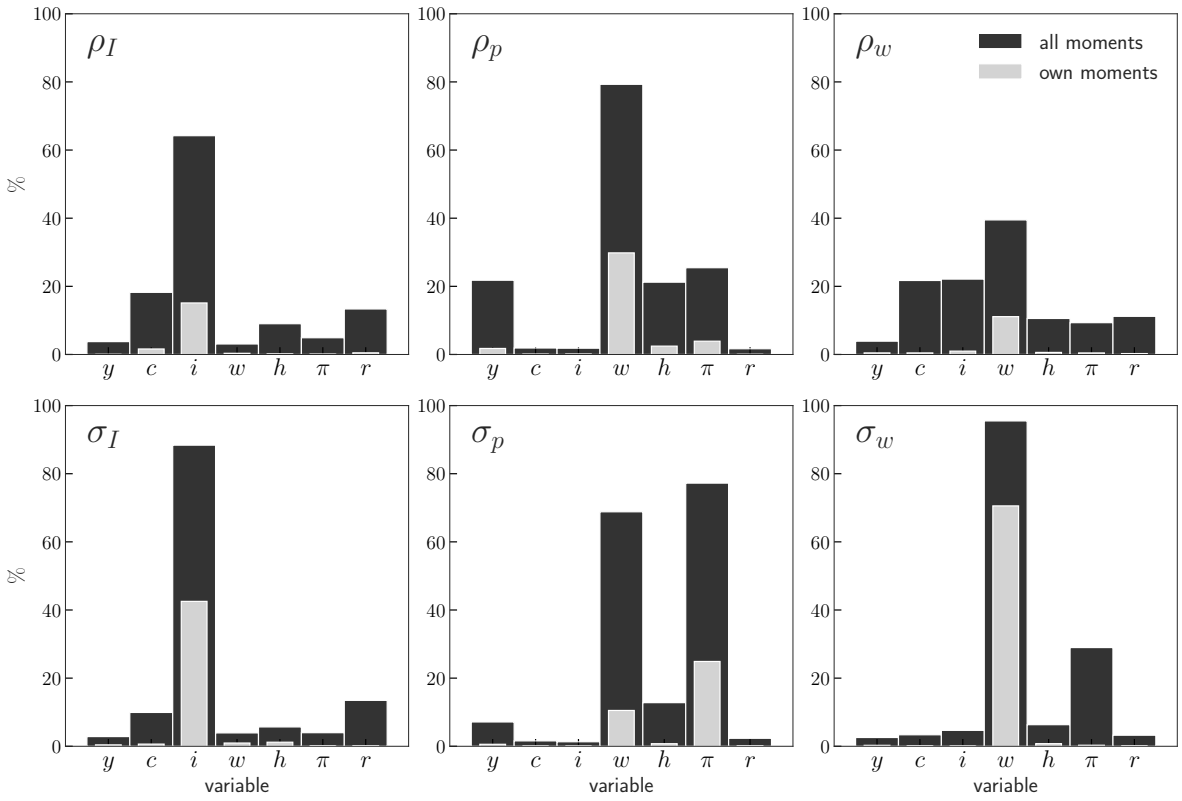

Figure B10: Investment-specific, price and wage markup shock parameters. The figure shows the efficiency gains due to moments of each observed variable.

B.3 Main groups of covariances

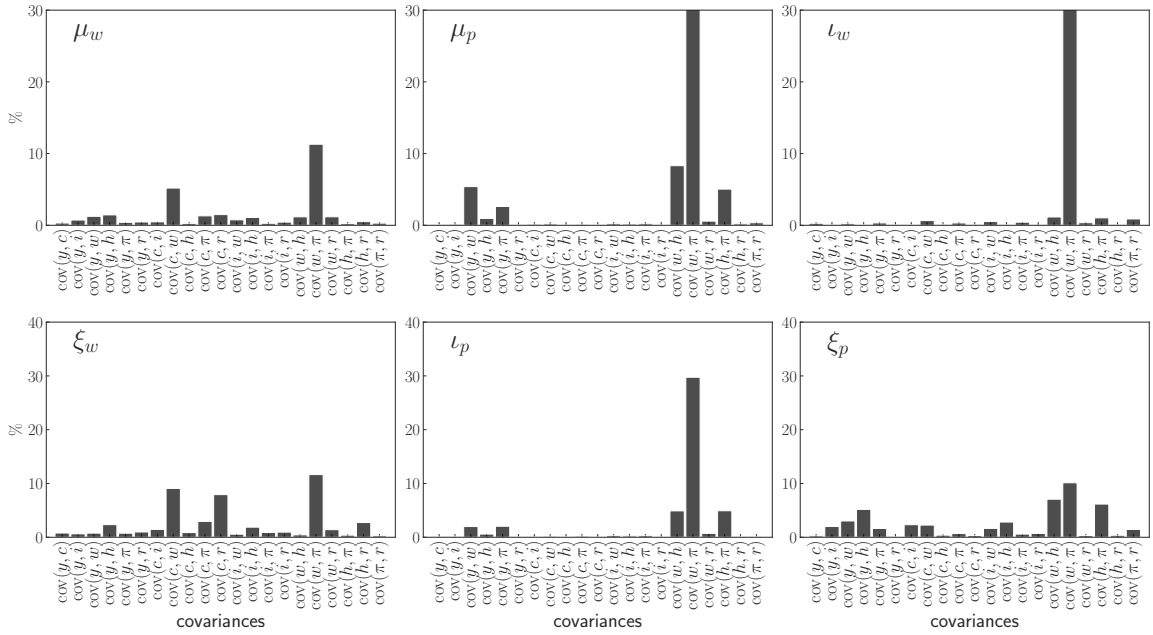

Figure B11: Price-wage block parameters. The figure shows the efficiency gains due the covariances in each group.

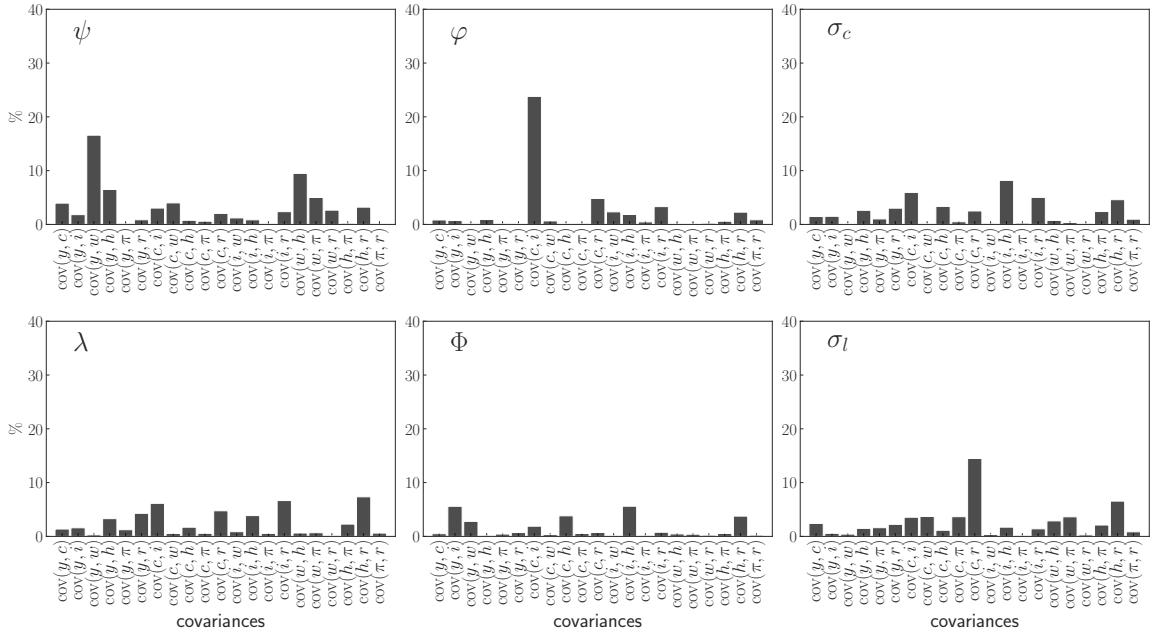

Figure B12: Preference and technology parameters. The figure shows the efficiency gains due the covariances in each group.

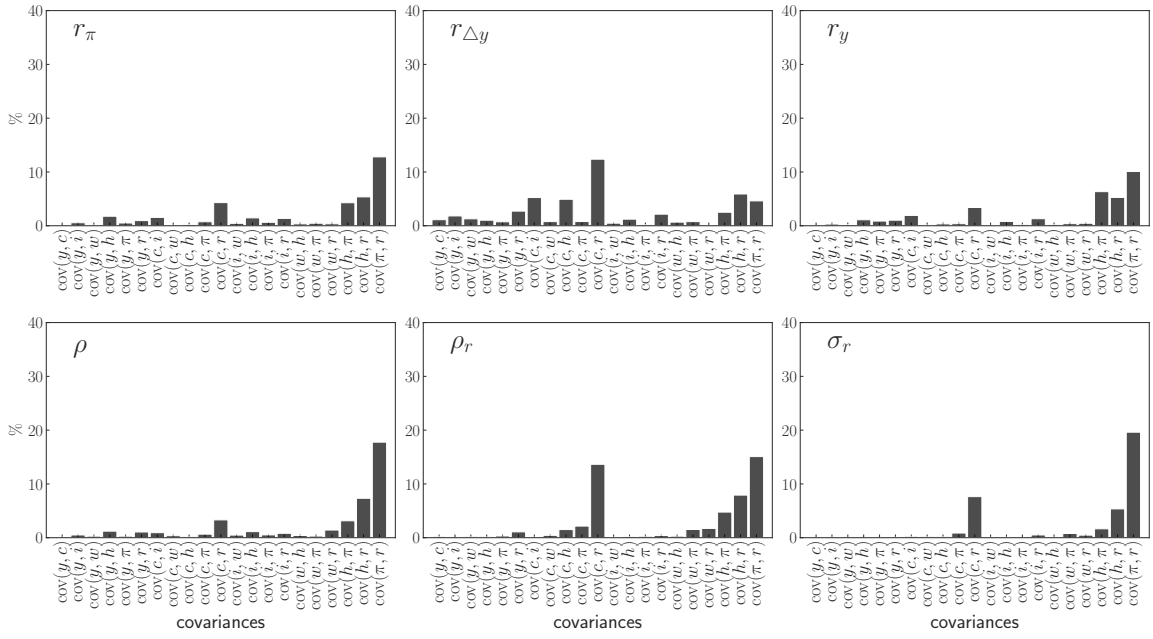

Figure B13: Monetary policy parameters. The figure shows the efficiency gains due the covariances in each group.

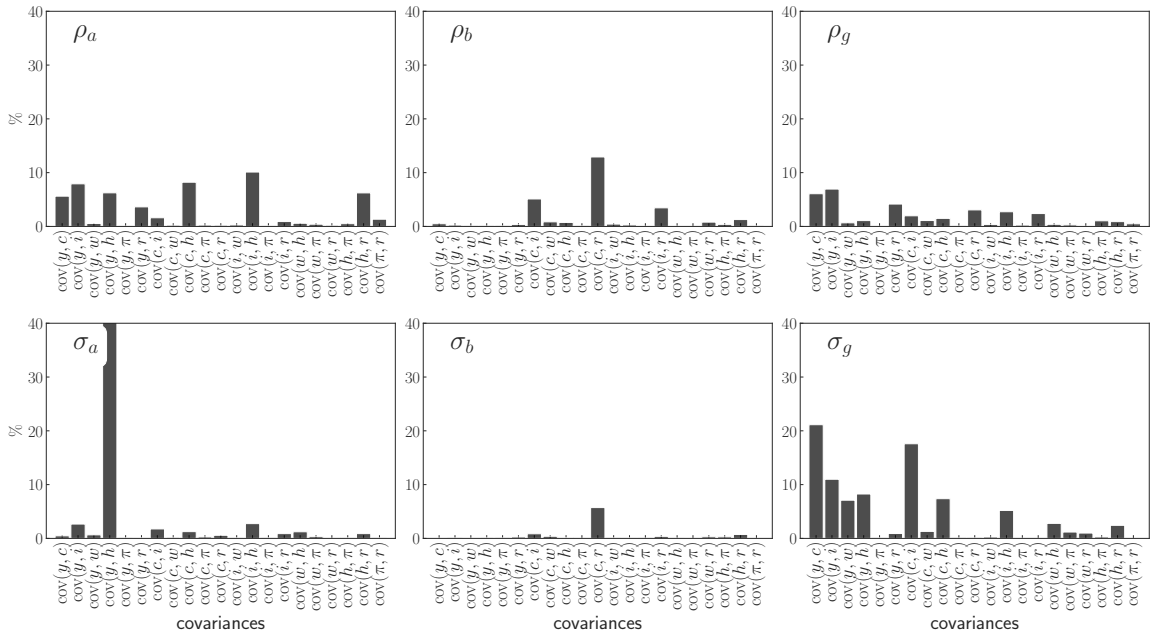

Figure B14: Productivity, risk premium, government spending shock parameters. The figure shows the efficiency gains due the covariances in each group.

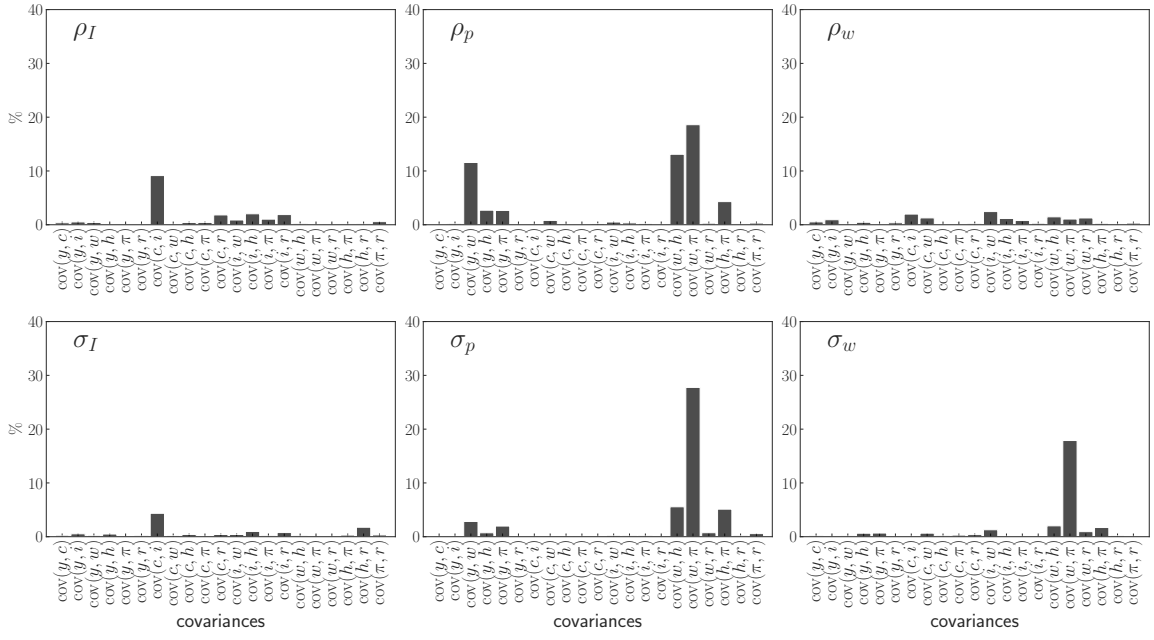

Figure B15: Investment-specific, price and wage markup shock parameters. The figure shows the efficiency gains due the covariances in each group.

B.4 Individual moments

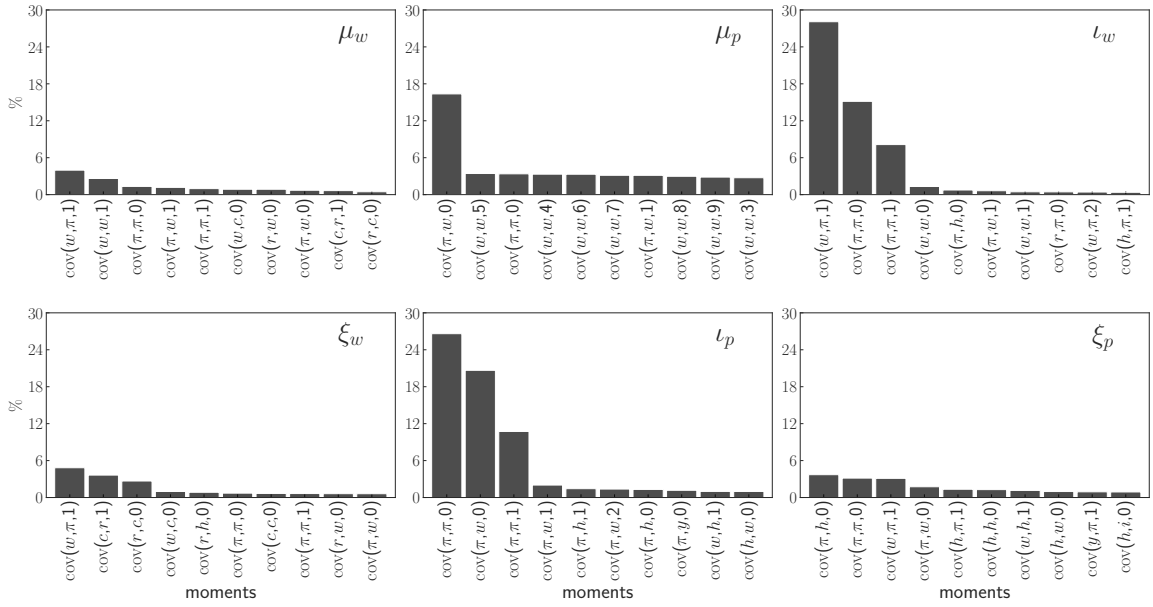

Figure B16: Price-wage block parameters. The figure shows the 10 most informative moments with respect to each parameter.

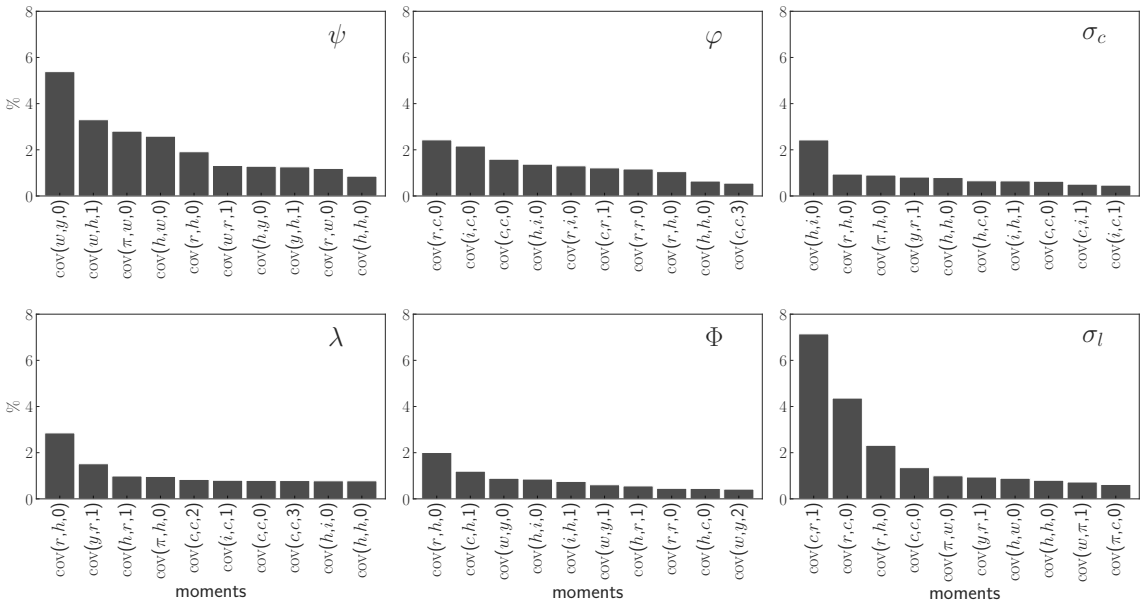

Figure B17: Preference and technology parameters. The figure shows the 10 most informative moments with respect to each parameter.

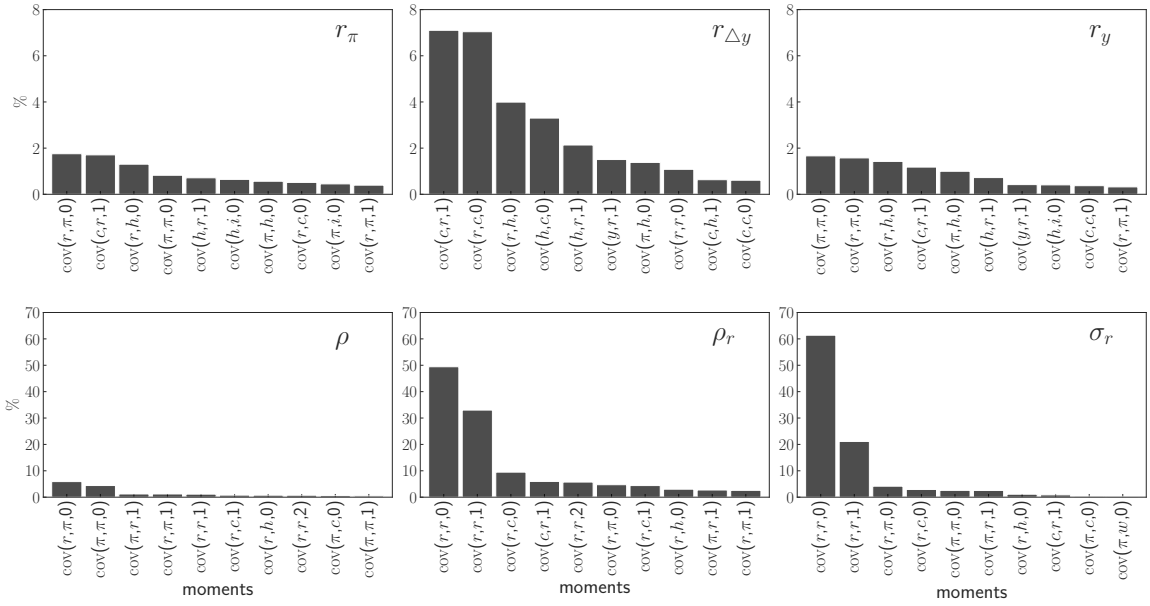

Figure B18: Monetary policy parameters. The figure shows the 10 most informative moments with respect to each parameter.

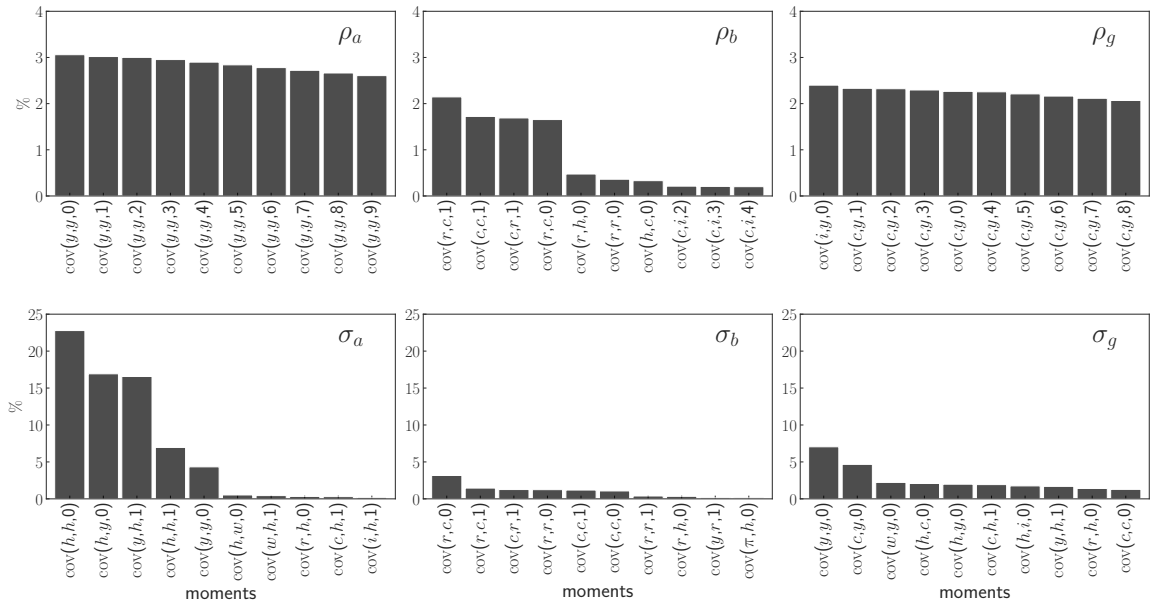

Figure B19: Productivity, risk premium, government spending shock parameters. The figure shows the 10 most informative moments with respect to each parameter.

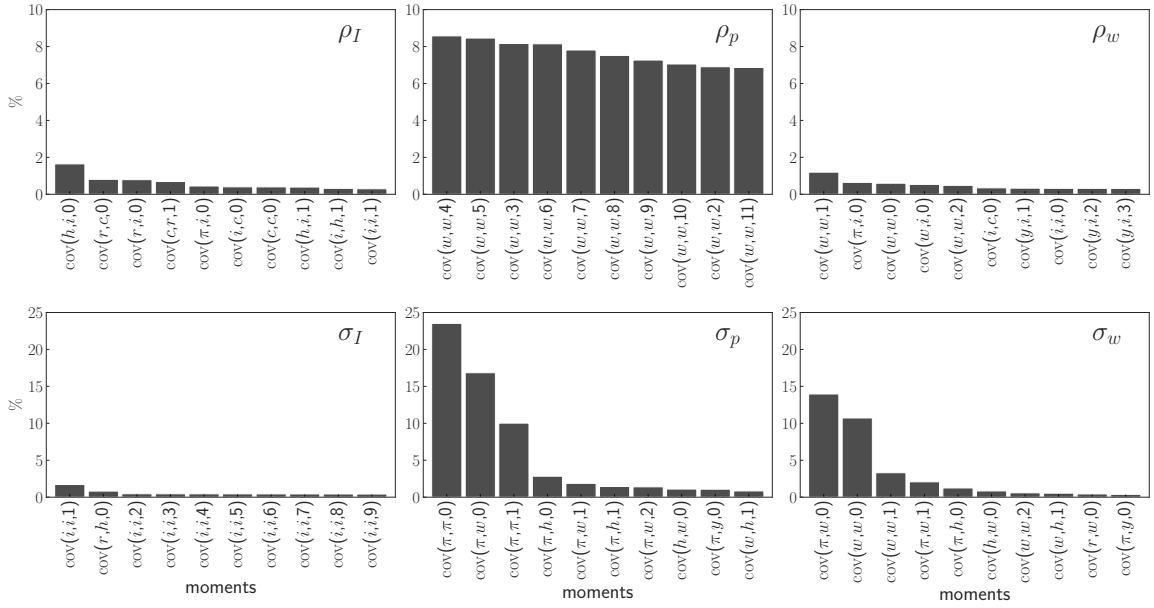

Figure B20: Investment-specific, price and wage markup shock parameters. The figure shows the 10 most informative moments with respect to each parameter.

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Schmitt-Grohé, S. and M. Uribe (2012): "What's News in Business Cycles," *Econometrica*, 80, 2733–2764.

Smets, F. and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *The American Economic Review*, 97, 586–606.