SPECTRAL DECOMPOSITION OF THE INFORMATION ABOUT LATENT VARIABLES IN DYNAMIC MACROECONOMIC MODELS

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The views expressed do not necessarily reflect the position of Banco de Portugal or the Eurosystem.

What?

What?

- spectral decomposition of information

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- where, in the spectrum, does information about latent variables come from

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- spectral decomposition of information
- where, in the spectrum, does information about latent variables come from
  - low, business cycle, high frequencies

Why?

Why?

- improve transparency

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- structural estimation: a black box

Why?

- improve transparency
- structural estimation: a black box
- increase credibility

SOME DEFINITIONS

LATENT VARIABLE

a variable treated as unobserved when a model is estimated

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    - potential output / output gap
    - natural rates
    - expectations
    - shocks, etc.

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a variable treated as unobserved when a model is estimated

- no data is available (abstract concepts)
  - potential output / output gap
  - natural rates
  - expectations
  - shocks, etc.
- available data is left out
  - stochastic singularity

MODEL

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$$F(y, x)$$

y - observed, x - latent

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$$F(x|y)$$

describes our knowledge of x given y and the model

INFORMATION

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- **unconditional**: information in y about x

$$\mathrm{var}\,(x) - \mathrm{var}(x|y)$$

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- **unconditional**: information in y about x

$$\text{var}\,(x) - \text{var}(x|y)$$

- **conditional**: information in y_1 about x given y_2

$$\text{var}\,(x|y_2) - \text{var}\,(x|y_1, y_2)$$

INFORMATION IN THE FREQUENCY DOMAIN

INFORMATION IN THE FREQUENCY DOMAIN

- decompose variance into frequency components
- information at frequency ω is the reduction of the variance at ω

INFORMATION GAIN MEASURES

UNCONDITIONAL INFORMATION GAIN

$$\mathrm{IG}_{\boldsymbol{y}\to x}(\omega) = \left(\frac{f_{xx}(\omega) - f_{x|\boldsymbol{y}}(\omega)}{f_{xx}(\omega)}\right) \times 100$$

$f_{xx}(\omega)$ - spectral density of x at ω

$f_{x|y}(\omega)$ - conditional spectral density of x given y at ω

- information in $\boldsymbol{y}$ about x

CONDITIONAL INFORMATION GAIN

$$\mathrm{IG}_{\boldsymbol{y}_1 \to x | \boldsymbol{y}_2}(\omega) = \left(\frac{f_{x|\boldsymbol{y}_2}(\omega) - f_{x|\boldsymbol{y}}(\omega)}{f_{xx}(\omega)} \right) \times 100$$

- information in $\boldsymbol{y}_1$ about x **given** $\boldsymbol{y}_2$

INTEGRATED INFORMATION GAIN

$$\mathrm{IG}_{\boldsymbol{y}\to x}(\boldsymbol{\omega}) = \int_{\omega \in \boldsymbol{\omega}} \mathrm{IG}_{\boldsymbol{y}\to x}(\omega) \frac{f_{xx}(\omega)}{f_{xx}(\boldsymbol{\omega})} \mathrm{d}\omega$$

$$\boldsymbol{\omega} = \{\omega : \omega \in [\underline{\omega}, \overline{\omega}] \cup [-\overline{\omega}, -\underline{\omega}]\}$$

INFORMATION GAIN DECOMPOSITION

$$\mathrm{IG}_{\boldsymbol{y}\to x}(\overline{\boldsymbol{\omega}}) = \mathrm{IG}_{\boldsymbol{y}\to x}(\boldsymbol{\omega}^L)\frac{f_{xx}(\boldsymbol{\omega}^L)}{f_{xx}(\overline{\boldsymbol{\omega}})} + \mathrm{IG}_{\boldsymbol{y}\to x}(\boldsymbol{\omega}^{BC})\frac{f_{xx}(\boldsymbol{\omega}^{BC})}{f_{xx}(\overline{\boldsymbol{\omega}})} + \mathrm{IG}_{\boldsymbol{y}\to x}(\boldsymbol{\omega}^H)\frac{f_{xx}(\boldsymbol{\omega}^H)}{f_{xx}(\overline{\boldsymbol{\omega}})}$$

$\overline{\boldsymbol{\omega}}$ - full spectrum

$\boldsymbol{\omega}^L$ - low frequencies

$\boldsymbol{\omega}^{BC}$ - BC frequencies (6 - 32 quarters)

$\boldsymbol{\omega}^H$ - high frequencies

APPLICATION

The neo-Fisher effect: Econometric evidence from empirical and optimizing models

Martín Uribe, AEJ: Macroeconomics (2022)

The neo-Fisher effect: Econometric evidence from empirical and optimizing models

Martín Uribe, AEJ: Macroeconomics (2022)

- small-scale New Keynesian model
- 7 shocks:
  - 3 monetary policy shocks
  - 2 preference shocks
  - 2 productivity shocks

MONETARY POLICY RULE

$$\frac{1+I_t}{\Gamma_t} = \left[A\left(\frac{1+\Pi_t}{\Gamma_t}\right)^{\alpha_t}\left(\frac{Y_t}{X_t}\right)^{\alpha_y}\right]^{1-\gamma_I}\left(\frac{1+I_{t-1}}{\Gamma_{t-1}}\right)^{\gamma_I} e^{z_t^m},$$

- Γ_t - inflation target
- X_t - non-stationary productivity
- z_t^m - stationary policy shock

INFLATION TARGET

$$\Gamma_t = X_t^m e^{z_t^{m2}},$$

- X_t^m - permanent component, grows at rate g_t^m
- z_t^{m2} - transitory component

PREFERENCE SHOCKS

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left\{ \frac{\left[\left(C_t - \delta \tilde{C}_{t-1} \right) \left(1 - e^{\theta_t} h_t \right)^{\chi} \right]^{1-\sigma} - 1}{1 - \sigma} \right\},$$

PRODUCTIVITY SHOCKS

$$Y_t = e^{z_t} X_t h_t^{\alpha},$$

- z_t - stationary productivity
- X_t - non-stationary productivity, grows at rate g_t

OBSERVED VARIABLES

- output growth (Δy_t)
- interest rate-inflation differential ($r_t = i_t - \pi_t$)
- change in the nominal interest rate (Δi_t)

all observed with measurement errors

INFORMATION ABOUT SHOCKS

SPECTRAL DECOMPOSITION

Table: Information gain decomposition across frequency bands

	total	low	BC	high
preference	93.2	70.4	19.5	3.2
labor supply	1.8	0.2	1.1	0.5
transitory productivity	1.8	0.2	1.1	0.5
permanent productivity	83.5	9.3	32.3	42.0
transitory interest rate	15.5	0.1	3.2	12.2
transitory trend inflation	16.5	5.8	9.7	1.0
permanent trend inflation	18.0	7.2	7.0	3.9

total = low + BC + high
(% of prior variance)

Table: Information gain decomposition across frequency bands

	total	low					BC					high				
preference	93.2	70.4	=	**96.4**	×	.73	19.5	=	**88.4**	×	.22	3.2	=	**66.0**	×	.05
labor supply	1.8	0.2	=	**0.5**	×	.33	1.1	=	**2.3**	×	.48	0.5	=	**2.9**	×	.18
transitory productivity	1.8	0.2	=	**0.5**	×	.32	1.1	=	**2.2**	×	.49	0.5	=	**2.9**	×	.19
permanent productivity	83.5	9.3	=	**94.9**	×	.10	32.3	=	**87.1**	×	.37	42.0	=	**78.9**	×	.53
transitory interest rate	15.5	0.1	=	**0.9**	×	.12	3.2	=	**7.9**	×	.41	12.2	=	**25.7**	×	.47
transitory trend inflation	16.5	5.8	=	**12.7**	×	.46	9.7	=	**23.1**	×	.42	1.0	=	**8.3**	×	.12
permanent trend inflation	18.0	7.2	=	**69.4**	×	.10	7.0	=	**18.3**	×	.38	3.9	=	**7.5**	×	.51

$$\text{contribution (band)} = \textbf{IG (band)} \times \frac{\text{variance (band)}}{\text{variance(total)}}$$

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	total	low					BC					high				
preference	93.2	70.4	=	96.4	×	**.73**	19.5	=	88.4	×	**.22**	3.2	=	66.0	×	**.05**
labor supply	1.8	0.2	=	0.5	×	**.33**	1.1	=	2.3	×	**.48**	0.5	=	2.9	×	**.18**
transitory productivity	1.8	0.2	=	0.5	×	**.32**	1.1	=	2.2	×	**.49**	0.5	=	2.9	×	**.19**
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transitory trend inflation	16.5	5.8	=	12.7	×	**.46**	9.7	=	23.1	×	**.42**	1.0	=	8.3	×	**.12**
permanent trend inflation	18.0	7.2	=	69.4	×	**.10**	7.0	=	18.3	×	**.38**	3.9	=	7.5	×	**.51**

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CONTRIBUTIONS BY OBSERVABLES

Table: Total information gains

shock	unconditional			conditional		
	$\triangle y_t$	r_t	$\triangle i_t$	$\triangle y_t$	r_t	$\triangle i_t$
preference	3.5	84.6	66.0	0.3	26.8	7.2
labor supply	0.0	0.6	1.7	0.1	0.1	1.1
transitory productivity	0.0	0.6	1.6	0.1	0.0	1.1
permanent productivity	76.7	0.1	0.1	83.4	0.8	5.7
transitory interest rate	0.7	5.8	11.5	2.2	1.5	9.0
transitory trend inflation	2.2	5.3	0.9	1.7	13.0	8.2
permanent trend inflation	1.8	0.4	6.8	0.5	10.4	15.6

INFORMATION COMPLEMENTARITY

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$$\mathrm{IC}_{\boldsymbol{y}_{12}\to x|\boldsymbol{y}_3}(\boldsymbol{\omega}) = \frac{\mathrm{IG}_{\boldsymbol{y}_{12}\to x|\boldsymbol{y}_3}(\boldsymbol{\omega})}{\mathrm{IG}_{y_1\to x|\boldsymbol{y}_3}(\boldsymbol{\omega}) + \mathrm{IG}_{y_2\to x|\boldsymbol{y}_3}(\boldsymbol{\omega})} - 1$$

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- positive: the contribution of each variable increases when the other is also observed

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- zero: the contribution doesn't depend on observing the other variable

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- zero: the contribution doesn't depend on observing the other variable

Note: total information doesn't decrease when complementarity is negative!

Information complementarity - Full spectrum

(a) Full spectrum unconditional

	ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m
$\Delta y_t,\ r_t$	-0.02	-0.01	-0.02	0.01	0.02	0.10	0.08
$\Delta y_t,\ \Delta i_t$	-0.05	0.03	0.02	0.08	0.16	0.14	-0.11
$r_t,\ \Delta i_t$	-0.38	-0.26	-0.26	-0.26	-0.23	1.37	1.43

(b) Full spectrum conditional

	ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m
$\Delta y_t,\ r_t$	-0.00	0.10	0.10	0.01	-0.08	-0.06	-0.03
$\Delta y_t,\ \Delta i_t$	-0.12	0.06	0.05	0.07	0.17	-0.10	-0.08
$r_t,\ \Delta i_t$	-0.38	-0.25	-0.25	-0.04	-0.23	0.91	1.51

Information complementarity - Low frequencies

(c) Low frequencies
unconditional

	ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m
Δy_t, r_t	-0.01	0.02	0.02	0.01	0.06	0.10	0.11
Δy_t, Δi_t	-0.02	0.04	0.04	0.03	0.14	-0.10	-0.02
r_t, Δi_t	-0.38	-0.42	-0.43	0.23	0.50	3.78	1.84

(d) Low frequencies
conditional

	ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m
Δy_t, r_t	-0.00	0.02	0.02	0.00	-0.02	-0.01	-0.00
Δy_t, Δi_t	-0.03	0.10	0.10	0.01	-0.06	-0.05	-0.01
r_t, Δi_t	-0.38	-0.42	-0.42	-0.29	0.40	3.32	1.86

Information complementarity - BC frequencies

(e) BC frequencies unconditional

	ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m
$\Delta y_t,\ r_t$	-0.08	-0.02	-0.03	0.03	0.05	0.12	0.12
$\Delta y_t,\ \Delta i_t$	-0.09	0.04	0.03	0.11	0.21	0.14	-0.20
$r_t,\ \Delta i_t$	-0.42	-0.31	-0.31	-0.31	-0.33	0.96	3.07

(f) BC frequencies conditional

	ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m
$\Delta y_t,\ r_t$	-0.01	0.13	0.13	0.02	-0.11	-0.08	-0.05
$\Delta y_t,\ \Delta i_t$	-0.12	0.11	0.11	0.09	0.16	-0.16	-0.12
$r_t,\ \Delta i_t$	-0.43	-0.29	-0.29	-0.07	-0.36	0.52	3.45

Information complementarity - High frequencies

(g) High frequencies unconditional

	ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m
Δy_t, r_t	-0.03	-0.03	-0.03	0.00	-0.00	0.01	0.02
Δy_t, Δi_t	-0.12	-0.01	-0.01	0.07	0.14	0.26	-0.11
r_t, Δi_t	-0.11	-0.06	-0.06	-0.22	-0.19	-0.25	0.18

(h) High frequencies conditional

ξ_t	θ_t	z_t	g_t	z_t^m	z_t^{m2}	g_t^m	
-0.03	0.09	0.09	0.00	-0.06	-0.07	-0.08	Δy_t, r_t
-0.13	-0.00	-0.01	0.07	0.17	0.19	-0.12	Δy_t, Δi_t
-0.12	-0.06	-0.06	0.03	-0.19	-0.28	0.17	r_t, Δi_t

CONCLUSION

What identifies (latent variable) x?

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- how much information?

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- main sources of information?

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What identifies (latent variable) x?

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- reveals if estimation uses data in ways that *may be unanticipated and undesired by other researchers and readers* (Andrews et al, 2020)
  - e.g. *is Uribe's model suitable for representing the very low/high frequencies in the data?*

From "***Quantifying confidence***" by Angeletos, Collard, and Dellas (2018)

*The models described above – like other business-cycle models – cater to business-cycle phenomena and therefore **omit** shocks and mechanisms that may account for medium- to long-run phenomena, such as trends in demographics and labor-market participation, structural transformation, regime changes in productivity growth or inflation, and so on.*

*In a nutshell, there is a risk of **contamination** of the estimates of a model by frequencies that the model was **not designed** to capture.*

There is nothing like a latent variable to stimulate the imagination
A. Goldberger, quoted by Chamberlain (1990)

FREQUENCY DOMAIN ANALYSIS OF TIME SERIES

Fact: any covariance stationary process can be written as:

$$Y_t = \int_0^{\pi} [a(\omega)\cos(\omega t) + b(\omega)\sin(\omega t)]d\omega$$

where $a(\omega)$ and $b(\omega)$ are independent random variables for all $\omega \in [0, \pi]$

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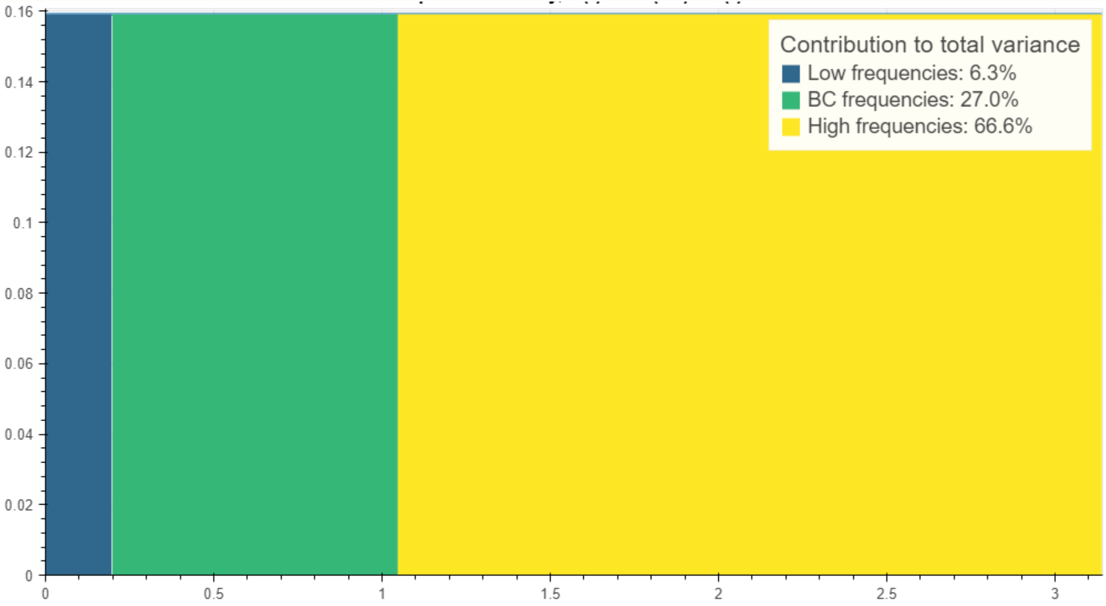
- $a(\omega)$ and $b(\omega)$ determine the contribution of ω for $\mathrm{var}(Y_t)$
- i.e. how important are ω cycles for Y_t
  - small $\omega \longrightarrow$ low frequency $\longrightarrow$ long (slow) cycles
  - large $\omega \longrightarrow$ high frequency $\longrightarrow$ short (fast) cycles

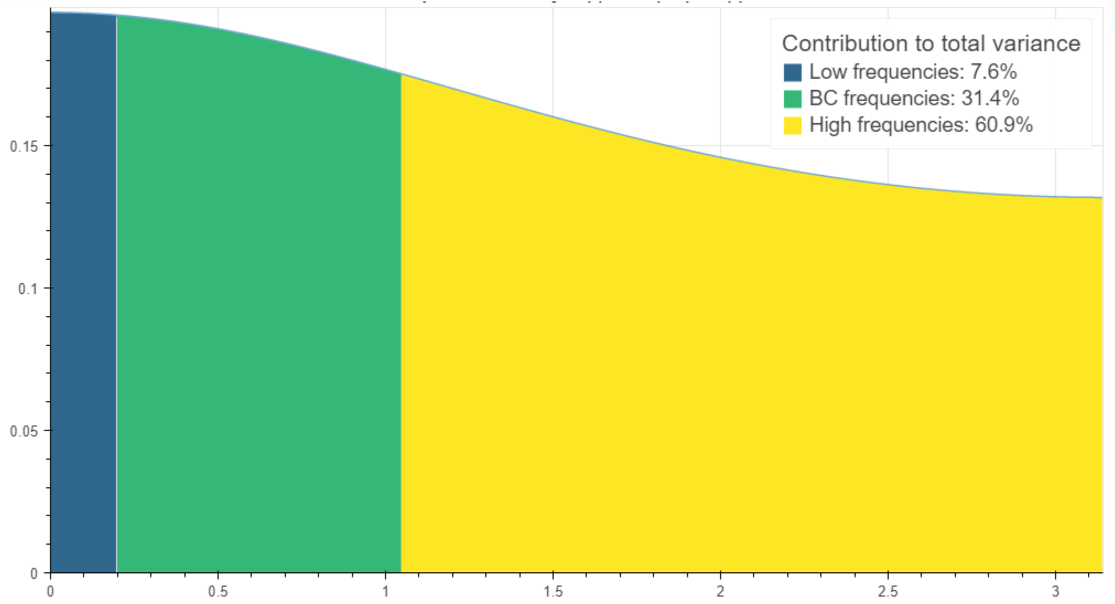
SPECTRAL DENSITY FUNCTION

spectral density of Y at $\omega \longrightarrow$ the contribution of ω to $\mathrm{var}(Y_t)$

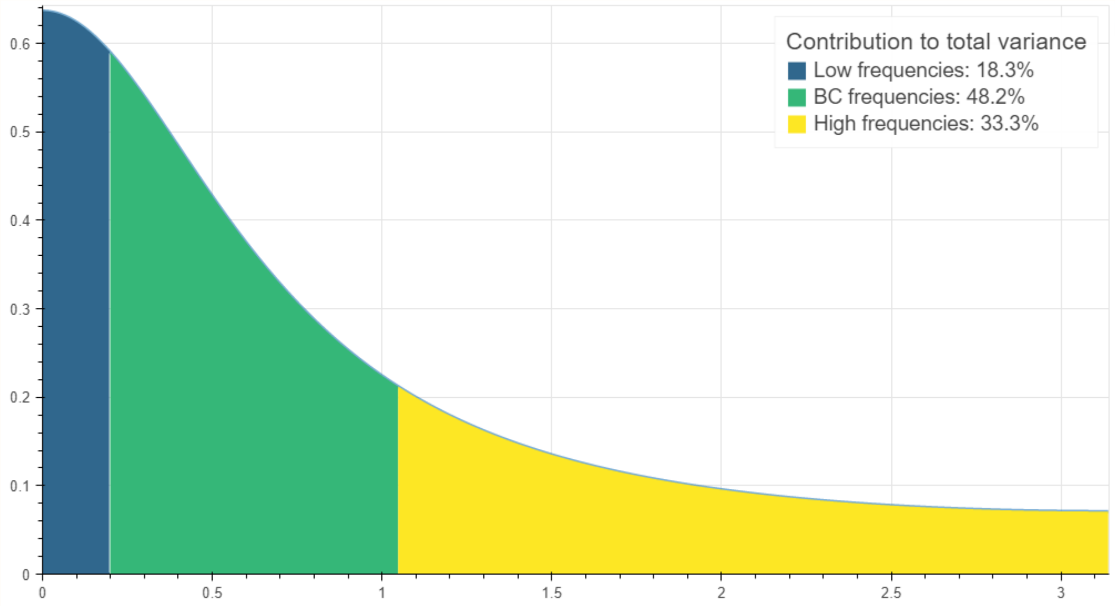
Spectral density of AR(1) process, $\alpha = 0$



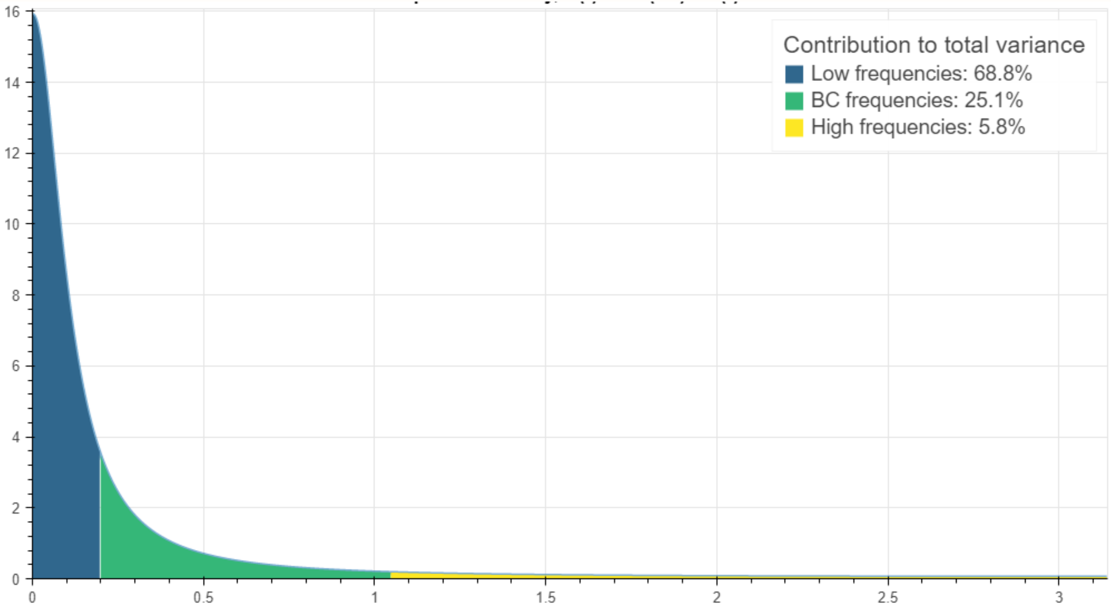
Spectral density of AR(1) process, $\alpha = 0.1$



Spectral density of AR(1) process, $\alpha = 0.5$



Spectral density of AR(1) process, $\alpha = 0.9$



INFORMATION GAINS IN THE TIME DOMAIN

$$\mathrm{IG}_{\boldsymbol{Y}_T \to x_t} = \left(\frac{\mathrm{var}(x_t) - \mathrm{var}(x_t | \boldsymbol{Y}_T)}{\mathrm{var}(x_t)} \right) \times 100,$$

where $1 \leq t \leq T$ and $\boldsymbol{Y}_T = \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T\}$.

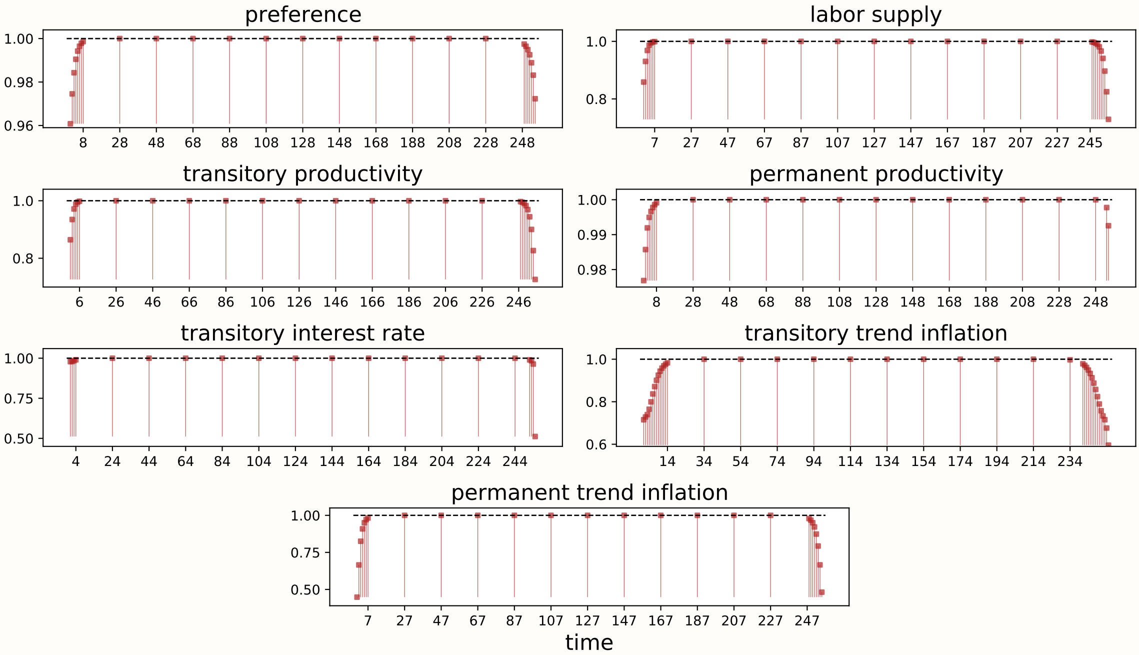

preference
1.00
0.98
0.96
8 28 48 68 88 108 128 148 168 188 208 228 248
labor supply
1.0
0.8
7 27 47 67 87 107 127 147 167 187 207 227 245
transitory productivity
1.0
0.8
6 26 46 66 86 106 126 146 166 186 206 226 246
permanent productivity
1.00
0.99
0.98
8 28 48 68 88 108 128 148 168 188 208 228 248
transitory interest rate
1.00
0.75
0.50
4 24 44 64 84 104 124 144 164 184 204 224 244
transitory trend inflation
1.0
0.8
0.6
14 34 54 74 94 114 134 154 174 194 214 234
permanent trend inflation
1.00
0.75
0.50
7 27 47 67 87 107 127 147 167 187 207 227 247
time

Table: Conditional information gains

shock	total			low			BC			high		
	$\triangle y_t$	r_t	$\triangle i_t$	$\triangle y_t$	r_t	$\triangle i_t$	$\triangle y_t$	r_t	$\triangle i_t$	$\triangle y_t$	r_t	$\triangle i_t$
preference	0.3	26.8	7.2	0.0	26.4	0.8	0.1	0.5	4.1	0.1	0.0	2.3
labor supply	0.1	0.1	1.1	0.0	0.0	0.0	0.1	0.0	0.6	0.0	0.0	0.5
transitory productivity	0.1	0.0	1.1	0.0	0.0	0.0	0.1	0.0	0.6	0.0	0.0	0.5
permanent productivity	83.4	0.8	5.7	9.3	0.0	0.1	32.2	0.6	2.8	41.9	0.2	2.8
transitory interest rate	2.2	1.5	9.0	0.0	0.1	0.0	0.4	0.9	0.4	1.8	0.5	8.5
transitory trend inflation	1.7	13.0	8.2	0.1	5.5	4.3	1.1	7.3	3.7	0.5	0.2	0.2
permanent trend inflation	0.5	10.4	15.6	0.0	4.7	7.0	0.2	5.2	5.6	0.3	0.5	3.0

LINEARIZED DSGE MODEL

$$
\begin{aligned}
\boldsymbol{y}_t &= \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{v}_{t-1} + \boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{u}_t \\
\boldsymbol{v}_t &= \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{v}_{t-1} + \boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u}_t \\
\boldsymbol{u}_t &= \boldsymbol{G}(\boldsymbol{\theta})\boldsymbol{u}_{t-1} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(\boldsymbol{\theta})\right)
\end{aligned}
$$

All model variables: $\boldsymbol{z}_t = [\boldsymbol{y}_t', \boldsymbol{v}_t', \boldsymbol{u}_t', \boldsymbol{\varepsilon}_t']'$

SPECTRAL DENSITY of $\boldsymbol{z}_t$

$$\boldsymbol{f}_{\boldsymbol{zz}}(\omega) = \frac{1}{2\pi}\boldsymbol{W}(\omega, \boldsymbol{\theta})\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(\boldsymbol{\theta})\boldsymbol{W}(\omega, \boldsymbol{\theta})^*$$

SPECTRAL DENSITY of z_t

$$\boldsymbol{f}_{\boldsymbol{zz}}(\omega) = \frac{1}{2\pi}\boldsymbol{W}(\omega,\boldsymbol{\theta})\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(\boldsymbol{\theta})\boldsymbol{W}(\omega,\boldsymbol{\theta})^*$$

where

$$\boldsymbol{W}(\omega,\boldsymbol{\theta}) = \begin{bmatrix} \boldsymbol{C}(\boldsymbol{\theta})e^{-i\omega} & \boldsymbol{D}(\boldsymbol{\theta}) & \mathbf{O}_{n_{\boldsymbol{y}},n_{\boldsymbol{u}}} \\ \boldsymbol{I}_{n_{\boldsymbol{v}}} & \mathbf{O}_{n_{\boldsymbol{v}},n_{\boldsymbol{u}}} & \mathbf{O}_{n_{\boldsymbol{v}},n_{\boldsymbol{u}}} \\ \mathbf{O}_{n_{\boldsymbol{u}},n_{\boldsymbol{v}}} & \boldsymbol{I}_{n_{\boldsymbol{u}}} & \mathbf{O}_{n_{\boldsymbol{v}},n_{\boldsymbol{u}}} \\ \mathbf{O}_{n_{\boldsymbol{u}},n_{\boldsymbol{y}}} & \mathbf{O}_{n_{\boldsymbol{u}},n_{\boldsymbol{u}}} & \boldsymbol{I}_{n_{\boldsymbol{u}}} \end{bmatrix} \times$$

$$\begin{bmatrix} \left(\boldsymbol{I}_{n_{\boldsymbol{v}}} - \boldsymbol{A}(\boldsymbol{\theta})e^{-i\omega}\right)^{-1}\boldsymbol{B}(\boldsymbol{\theta})\left(\boldsymbol{I}_{n_{\boldsymbol{u}}} - \boldsymbol{G}(\boldsymbol{\theta})e^{-i\omega}\right)^{-1} \\ \left(\boldsymbol{I}_{n_{\boldsymbol{u}}} - \boldsymbol{G}(\boldsymbol{\theta})e^{-i\omega}\right)^{-1} \\ \boldsymbol{I}_{n_{\boldsymbol{u}}} \end{bmatrix}$$

- spectral density of any latent variable x:

$$f_{xx}(\omega)$$

- conditional spectral density of x given any set of observed variables $\boldsymbol{y}$:

$$f_{x|\boldsymbol{y}}(\omega) = f_{xx}(\omega) - f_{x\boldsymbol{y}}(\omega) f_{\boldsymbol{yy}}^{-1}(\omega) f_{\boldsymbol{y}x}(\omega)$$